UNIVERSITY OF BIRMINGHAM | GRAVITATIONAL WAVE ASTRONOMY

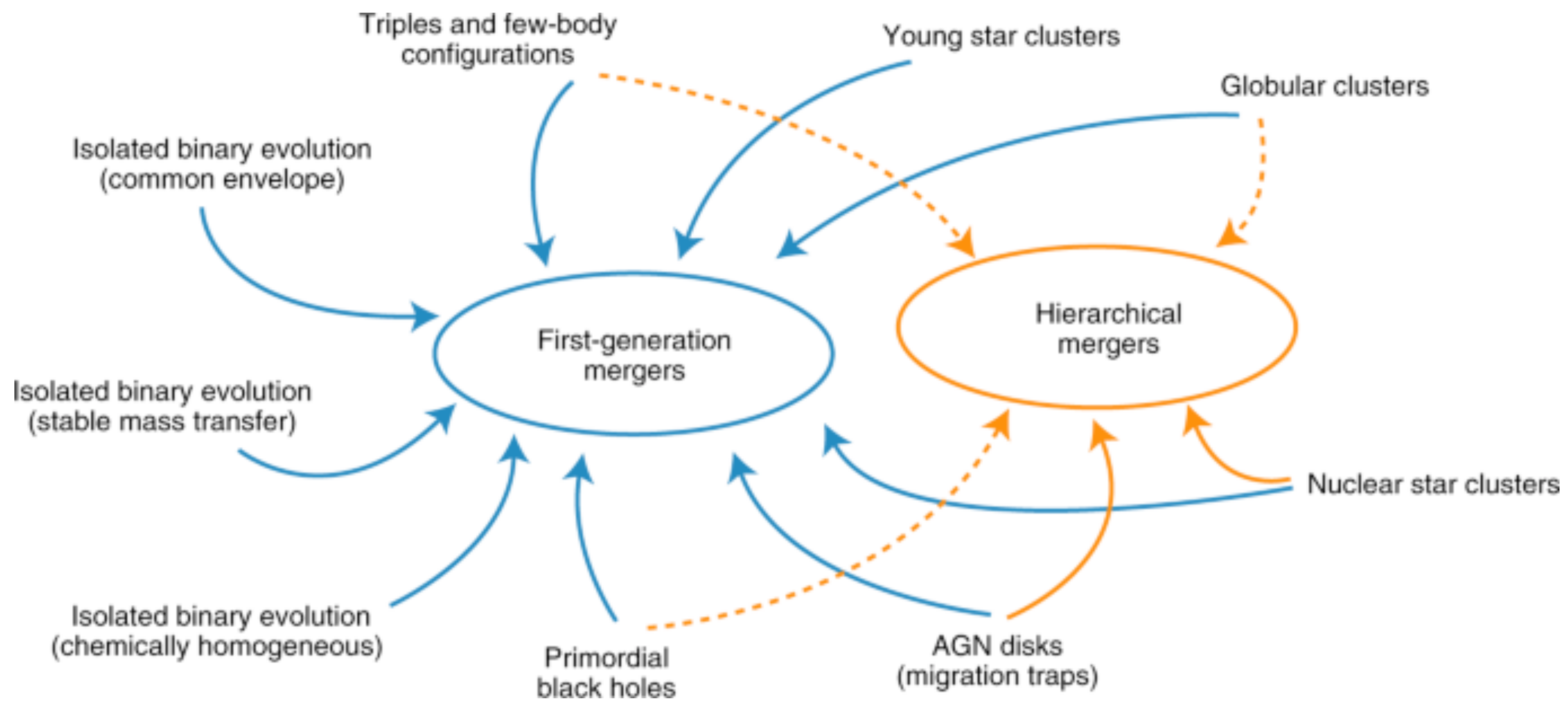
LECTURE 1: INTRODUCTION TO GRAVITATIONAL WAVES

PATRICIA SCHMIDT

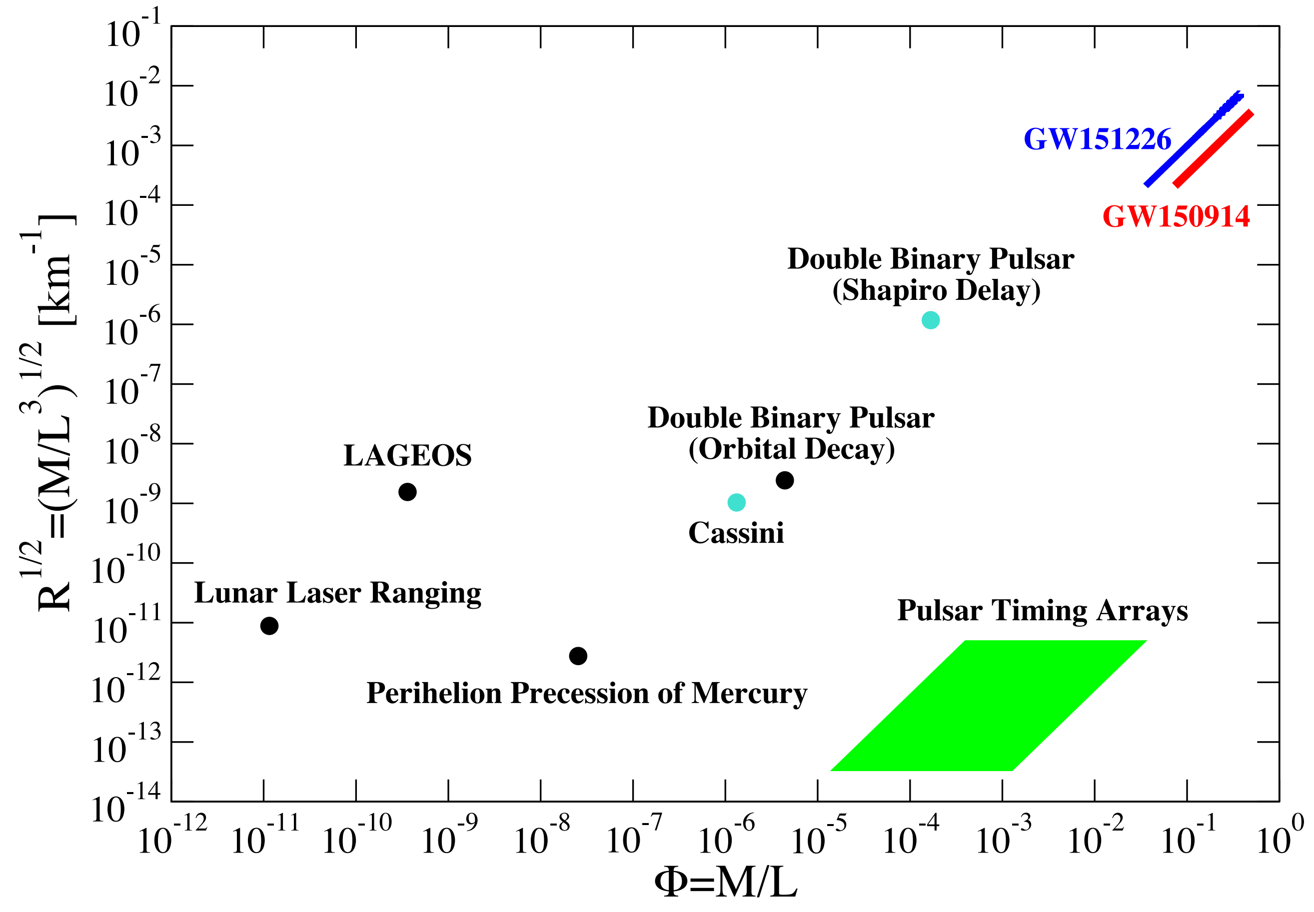
ISCRA 22

GRAVITATIONAL-WAVE SCIENCE

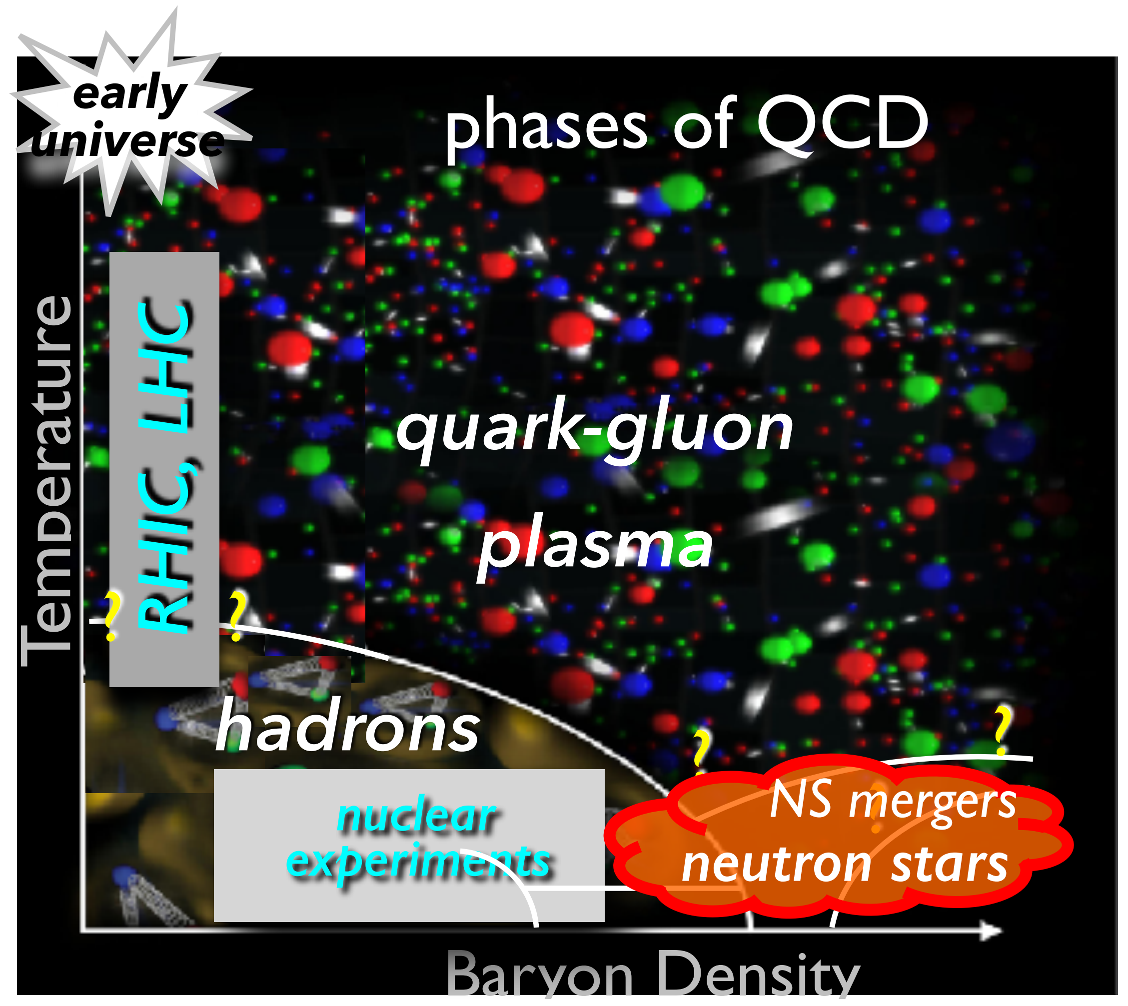
- Discover the dark side of the Universe
  - Detect and determine properties of astrophysical (and primordial?) black holes
  - Measure merger rates of compact binaries
  - Inform binary formation models
- Infer the EOS of matter at supra-nuclear densities e.g. in neutron stars
- Test GR in the strong-field, high-curvature regime
- Independently measure the expansion rate of the Universe
- Multimessenger astrophysics
- Dark energy, dark matter
- …



[Gerosa&Fishbach]



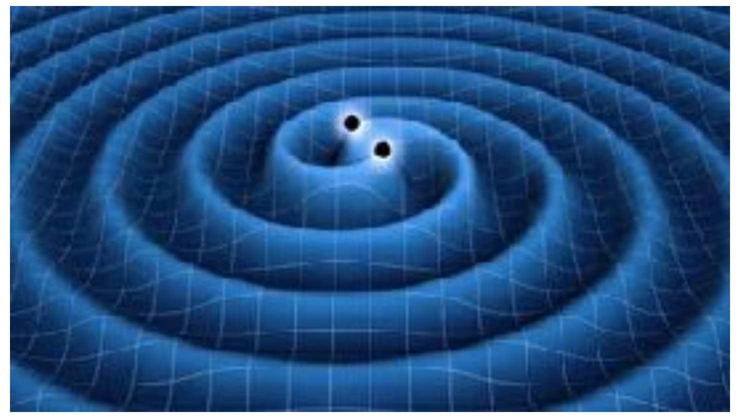
[Yunes, 2016]

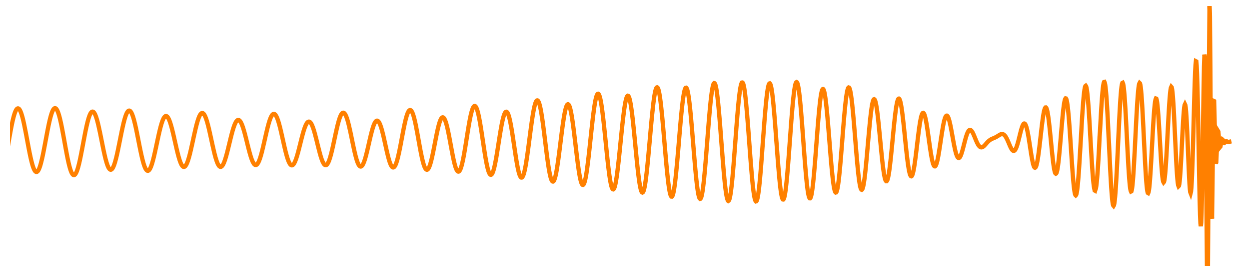


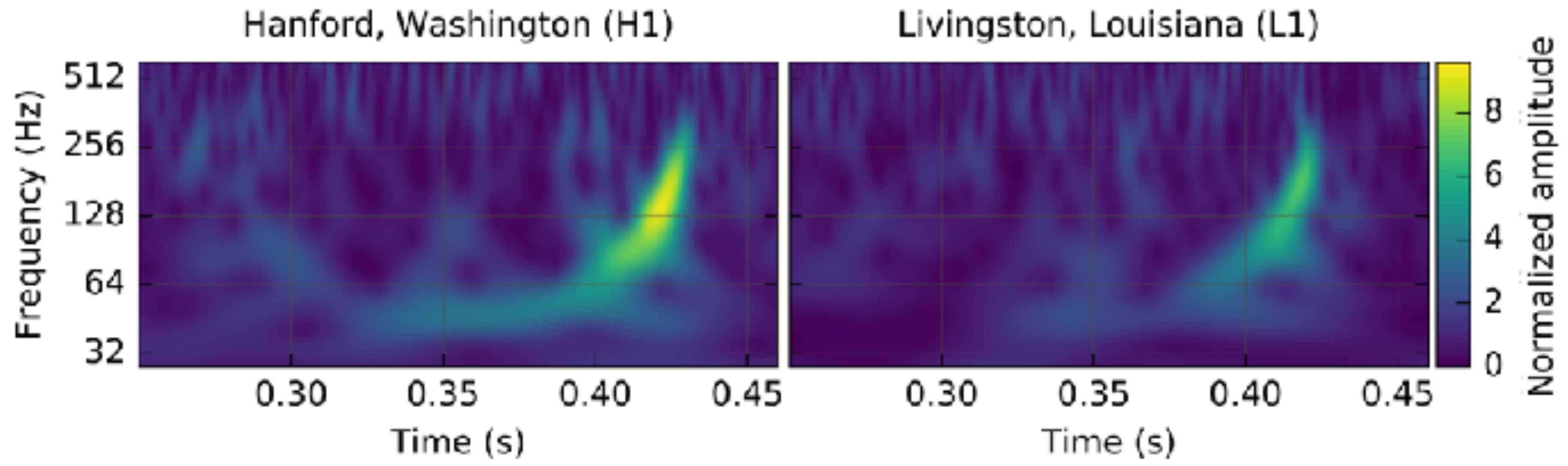
Credit: F. Linde

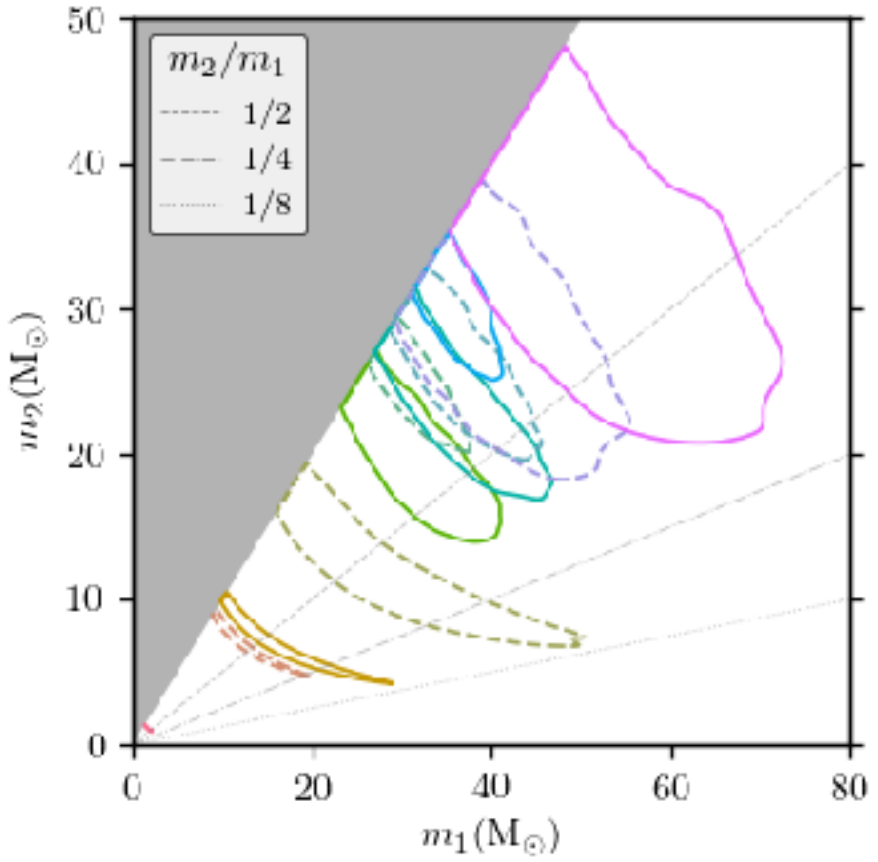
OUTLINE

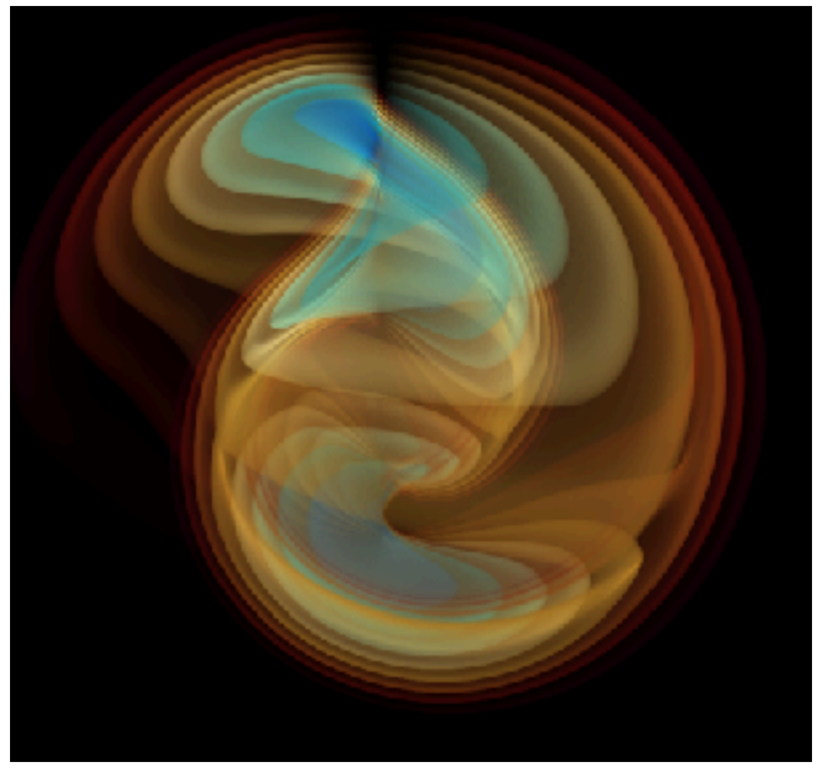
- **Lecture 1: Introduction; 4.8. @ 15.00**
    - What are gravitational waves?
    - Sources of gravitational waves
    - Gravitational-wave detectors
- **Lecture 2: Data Analysis for compact binaries; 5.8. @ 11.45**
    - Detection: Matched filtering
    - Parameter Estimation
    - Modelling gravitational waves from compact binaries
- **Lecture 3: Observations; 6.8. @ 9.00**
    - Gravitational-wave observations
    - Future missions and prospects





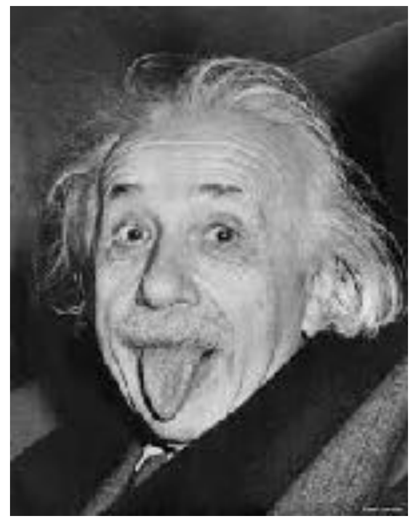






GR REMINDER

- The gravitational field is a geometric property of 4D spacetime: **curvature**
    - **Metric tensor** $g_{\mu\nu}$: how to measure distances and angles in a curved manifold
    - Mass/energy curve spacetime



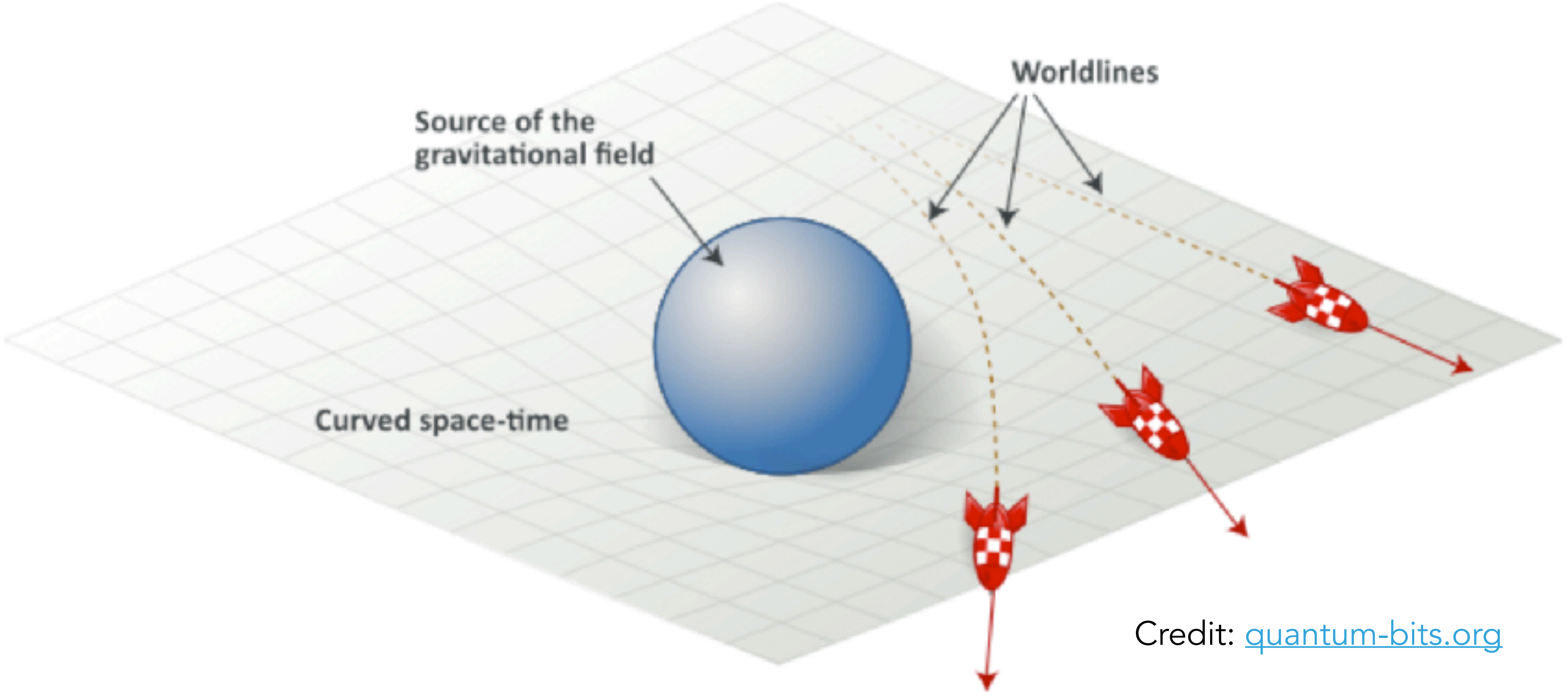
Conventions:

$$\mathrm{sign}(\eta_{\mu\nu}) = (-1,1,1,1)$$
$$u^\mu v_\mu = \sum_\mu u^\mu v_\mu$$
$$\mu \in \{0,1,2,3\}$$
$$i \in \{1,2,3\}$$
$$G = c = 1$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

Einstein field equations

- Locally, for freely-falling observers the laws of special relativity hold (equivalence principle)
    - Freely-falling observers move along **geodesics** (shortest paths in general manifolds)
    - **Tidal effects** determine the relative acceleration between 2 freely-falling observers



Credit: quantum-bits.org

LINEARISED GRAVITY

- Are a fundamental prediction of General Relativity (GR): propagating oscillations of the gravitation field generated by accelerating masses
  - Transverse waves travelling at the speed of light c
- Let us consider the vacuum Einstein field equations (far away from the source of the gravitational field):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 0$$

- **Linearised gravity:** Far away from the source of the gravitational field, the metric $g_{\mu\nu}$ is that of flat Minkowski space with a small metric perturbation $h_{\mu\nu}$, i.e.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad ||h_{\mu\nu}|| \ll 1$$

  - Compute all relevant quantities keeping only the terms linear in $h_{\mu\nu}$ (higher order terms are discarded)
  - Work with the trace-reversed metric perturbation to simplify expressions: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$

LINEARISED GRAVITY

- Make use of the gauge freedom in GR!
  - Using the Lorenz (harmonic) gauge, $\partial^\nu \bar{h}_{\mu\nu} = 0$, the Einstein field equations reduce to a wave equation for the trace-reversed metric perturbation tensor:

$$\Box \bar{h}_{\mu\nu} \equiv \eta_{\mu\nu}\partial^\mu \partial^\nu \bar{h}_{\mu\nu} = \left(-\frac{1}{c^2}\partial_t^2 + \nabla^2 \right) \bar{h}_{\mu\nu} = 0$$

flat-space d'Alembertian

- Solutions to the wave equation are (superpositions of) plane waves:

$$\bar{h}_{\mu\nu}(t;\vec{x}) = \mathrm{Re} \int d^3k A_{\mu\nu}(\vec{k}) e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

  - Note: $k^\mu = (\omega, \overrightarrow{k})$ and $k^\mu A_{\mu\nu} = 0$ because of the Lorenz gauge

These are gravitational waves!

TRANSVERSE-TRACELESS GAUGE

- The Lorenz gauge condition does not fix the GR gauge freedom completely for globally vacuum, asymptotically flat spacetimes
  - Impose 4 additional gauge conditions: $h = 0$ (traceless) & $h_{00} = h_{0i} = 0$ (purely spatial)
  - From the Lorenz gauge condition it follows that $\partial^i h_{ij} = 0$, i.e. the metric perturbation is transverse
  - This is the transverse-traceless (TT) gauge, which is not necessary but very convenient.
  - The remaining DOF contain only physical information, non-gauge information about GWs!
- For a plane-wave travelling along the z-axis, the metric perturbation tensor in the TT gauge becomes:

$$h_{ij}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos(\omega(t - z/c))$$

- 2 DOF: $h_+, h_\times$ are the two independent gravitational-wave polarisations
- Note: One can show that the radiative DOF are always contained in the TT-part of the metric perturbation in any gauge!

INTERACTION OF GWS WITH TEST MASSES

- In curved space, test masses move along **geodesics** parameterised by the proper time τ:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\mu}{d\tau}\frac{dx^\sigma}{d\tau} = 0$$

geodesic equation

- Let us consider two nearby geodesics, separated by an infinitesimal vector $\xi^\mu(\tau)$. If the separation is much smaller than the typical scale of the variation of the gravitational field, the first-order expansion leads to the **geodesic deviation equation**:

$$\frac{d^2\xi^\mu}{d\tau^2} + 2\Gamma^\mu_{\nu\sigma}\frac{dx^\mu}{d\tau}\frac{d\xi^\sigma}{d\tau} + \xi^\sigma\partial_\sigma\Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{d\xi^\rho}{d\tau} = 0$$

$$\frac{D^2\xi^\mu}{D\tau^2} = -R^\mu{}_{\nu\rho\sigma}\xi^\rho\frac{dx^\nu}{d\tau}\frac{dx^\sigma}{d\tau}$$

- The separation between the two geodesics changes with time in the presence of a gravitational field
    - Two nearby time-like geodesics experience a tidal force, which is determined by the Riemann tensor.

INTERACTION OF GWS WITH TEST MASSES

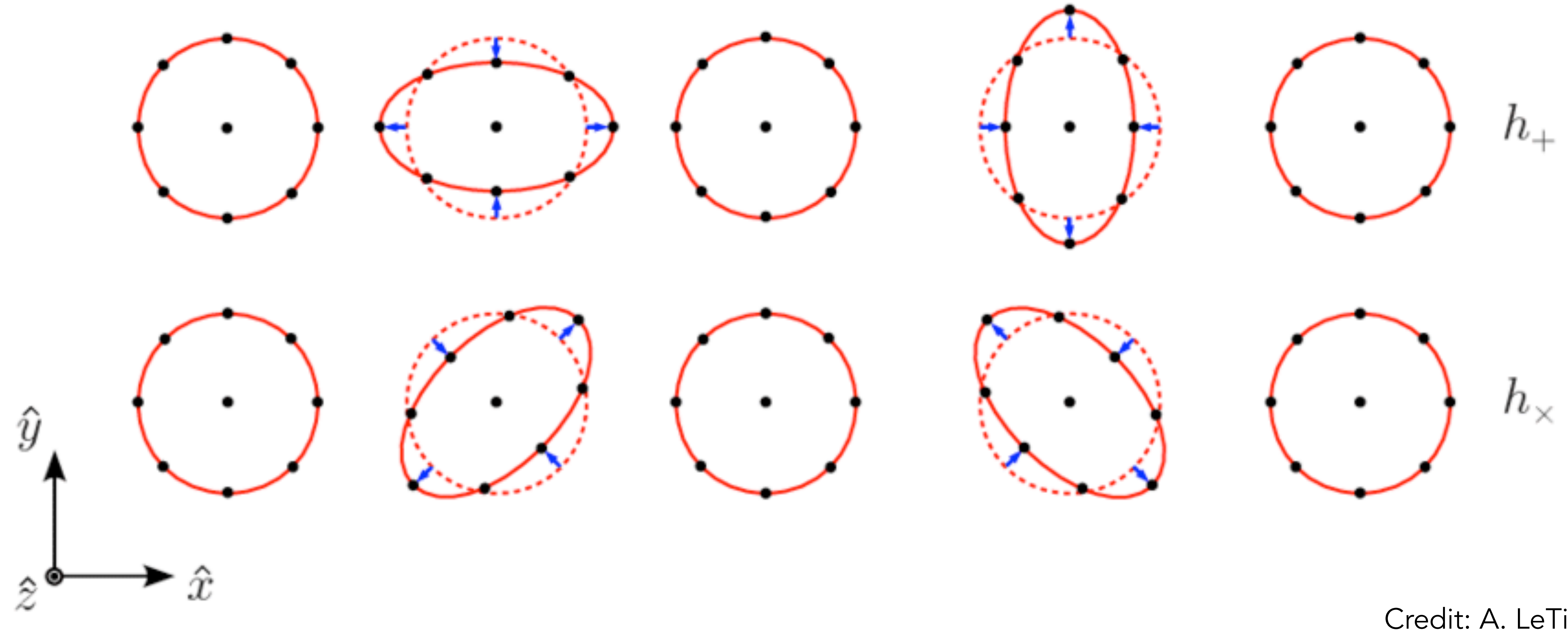
- Consider a local rest frame at a point P; i.e.
  $g_{\mu\nu}(P) = \eta_{\mu\nu} \quad \rightarrow \quad \Gamma^{\mu}{}_{\nu\sigma}(P) = 0$

- Consider a non-relativistic observer (e.g. a GW detector), then $dx^i/d\tau \ll dx^0/d\tau$

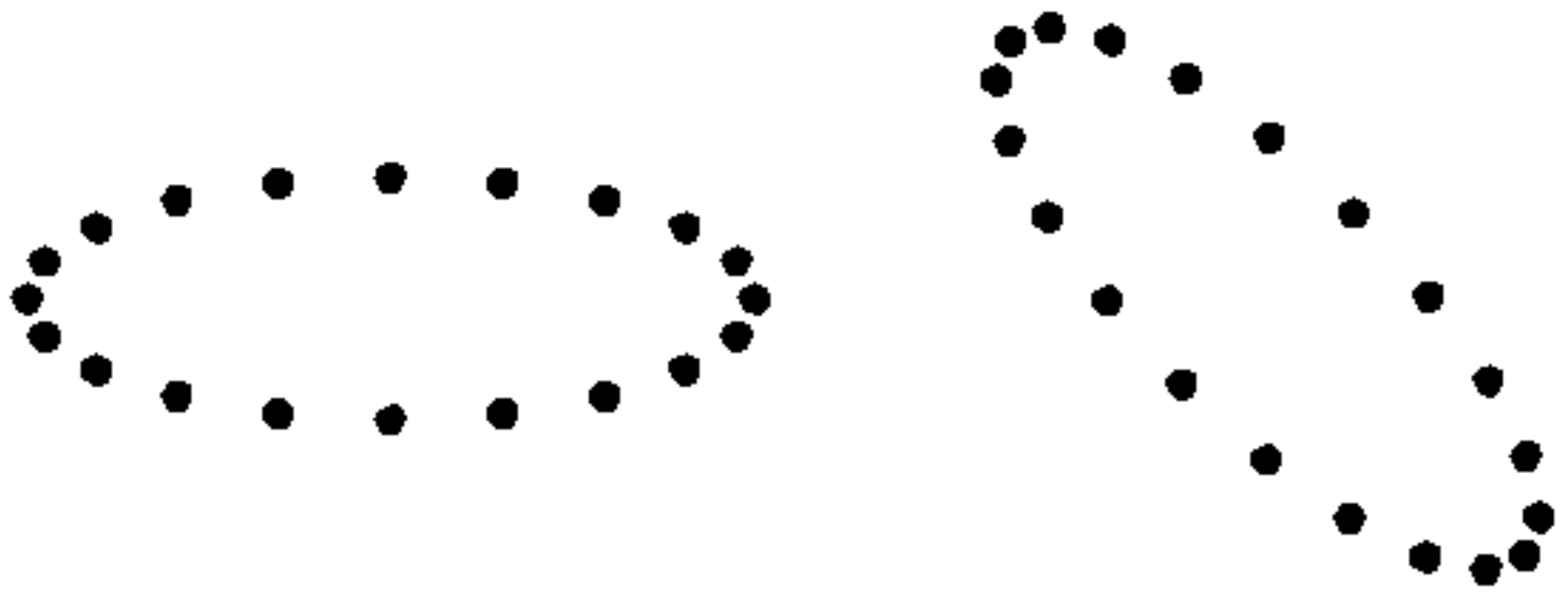
- Under these assumptions, the geodesic deviation equation reduces to:

$$\ddot{\xi}^i = \frac{1}{2}\ddot{h}^{\mathrm{TT}}_{ij}\xi^j$$

- Gravitational waves have the effect of tidal waves, i.e. they change the **proper separation** between two freely-falling test masses periodically: "stretching" and "squeezing" of spacetime



Credit: A. LeTiec



INTERACTION OF GWS WITH TEST MASSES

- Let us consider two freely falling test masses located at $z = 0$ and separated by a coordinate distance L_c along the x-axis.
- Consider a GW travelling down the z-axis in the TT gauge: $h_{\mu\nu}^{\mathrm{TT}}(t; z)$.
- Then the proper distance L between the two test masses is given by:

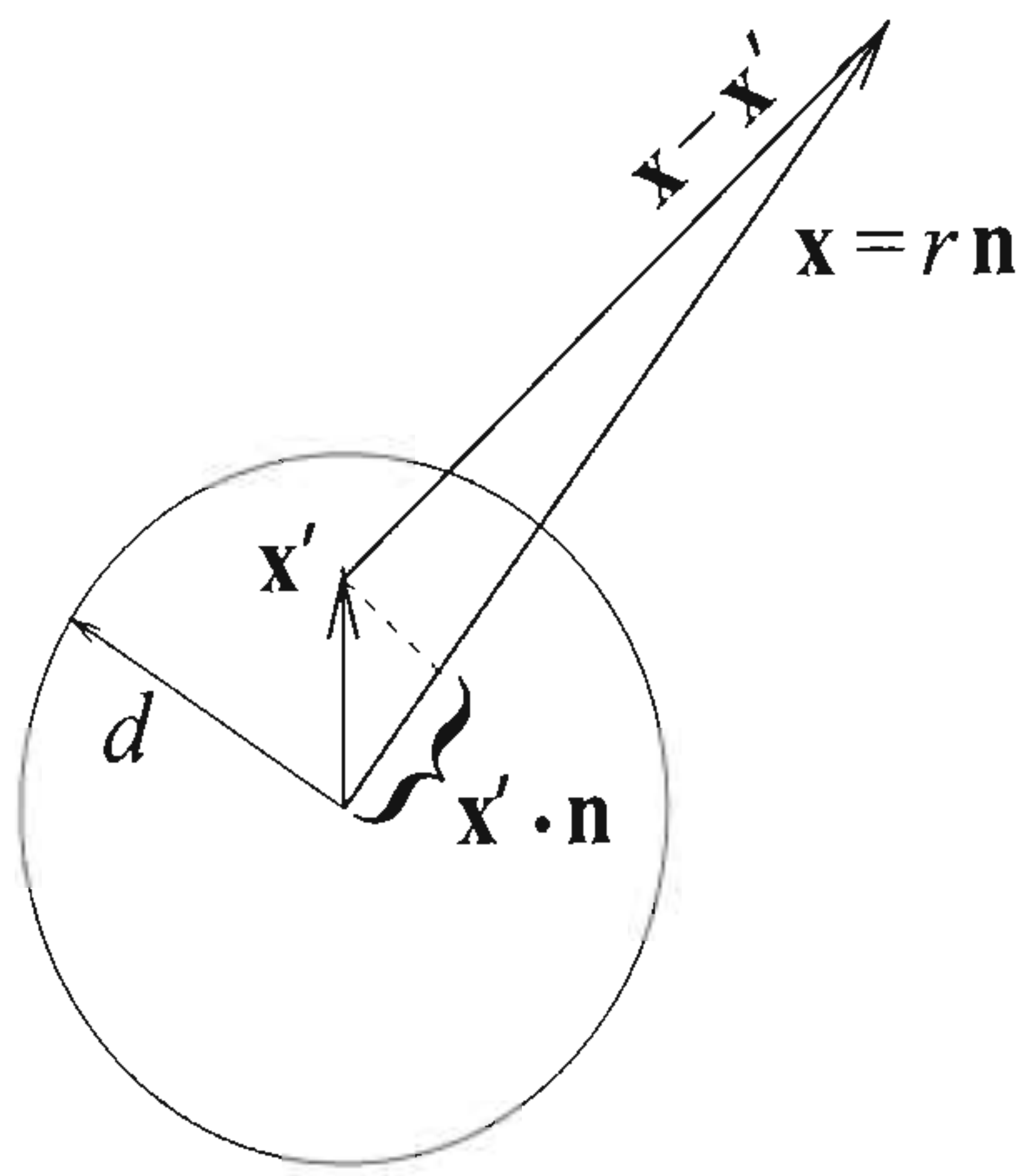
$$L = \int_0^{L_c} dx \sqrt{g_{xx}} = \int_0^{L_c} dx \sqrt{1 + h_{xx}^{\mathrm{TT}}(t; z=0)}$$
$$\simeq \int_0^{L_c} dx \left[1 + \frac{1}{2} h_{xx}^{\mathrm{TT}}(t; z=0)\right] = L_c \left[1 + \frac{1}{2} h_{xx}^{\mathrm{TT}}(t; z=0)\right]$$

  - Note: We used the fact that the coordinate separation remains fixed in the TT gauge.
- When a GW passes, the proper separation changes by a fractional length change (**strain**) $\delta L / L$ given by

$$\frac{\delta L}{L} \simeq \frac{1}{2} h_{xx}^{\mathrm{TT}}(t; z=0)$$

This fractional length change = strain is what we measure in GW detectors!

GENERATION OF GRAVITATIONAL WAVES: QUADRUPOLE FORMULA



- Let us assume a slowly moving source in linearised gravity: $v \ll c$
- The solutions to the inhomogeneous wave equation are plane waves (in the Lorenz gauge): $\bar{h}_{\mu\nu}(t;\vec{x}) = 4\int d^3x' \frac{T_{\mu\nu}(t-|\vec{x}-\vec{x}'|;\vec{x}')}{|\vec{x}-\vec{x}'|}$. Recall that the radiative degrees of freedom are contained in the spatial TT-part of the metric: $\mu\nu \rightarrow ij$
- At large distance from the source, we can perform a multipole expansion of the denominator analogous the EM to find $\bar{h}_{ij}(t;\vec{x}) = \frac{4}{r}\int d^3x' T_{ij}(t-r;\vec{x}')$ (at linear order), where $r := |\vec{x}|$.
- Using the continuity equation in linearised gravity, i.e. $\partial_\mu T^{\mu\nu} = 0$, we can further simplify this integral: $\frac{4}{r}\int d^3x' T_{ij}(t-r;\vec{x}') = \frac{2}{r}\frac{\partial^2}{\partial t^2}\int d^3x' \rho x'^i x'^j$. Using the definition of the moment of inertia tensor, we arrive at: $\bar{h}_{ij}(t;\vec{x}) = \frac{2}{r}\frac{d^2 I_{ij}(t-r)}{dt^2}$. By projecting out the TT part, we arrive at the final answer - the **quadrupole formula**:

$$h_{ij}^{\mathrm{TT}}(t;\vec{x}) = \frac{2}{r}\frac{d^2\mathcal{I}_{kl}(t-r)}{dt^2}P_{ik}(\hat{n})P_{jl}(\hat{n})$$

mass quadrupole

GENERATION OF GRAVITATIONAL WAVES: LUMINOSITY

- Gravitational waves are some of the most luminous events in the universe
  - GW150914 emitted about 3 solar masses in GWs!
- GW waves carry energy and linear momentum away from the source
- The **stress-energy tensor of a propagating gravitational field** is given by the Isaacson expression

$$T_{\mu\nu} = \frac{1}{32\pi}\langle h^{\mathrm{TT}}_{jk,\mu} h^{\mathrm{TT}jk}_{,\nu}\rangle$$

  - Brackets denote an average of regions of the size of the wavelength and times of the length of the period.
- The GW luminosity is obtained by integrating the flux over a distant sphere:

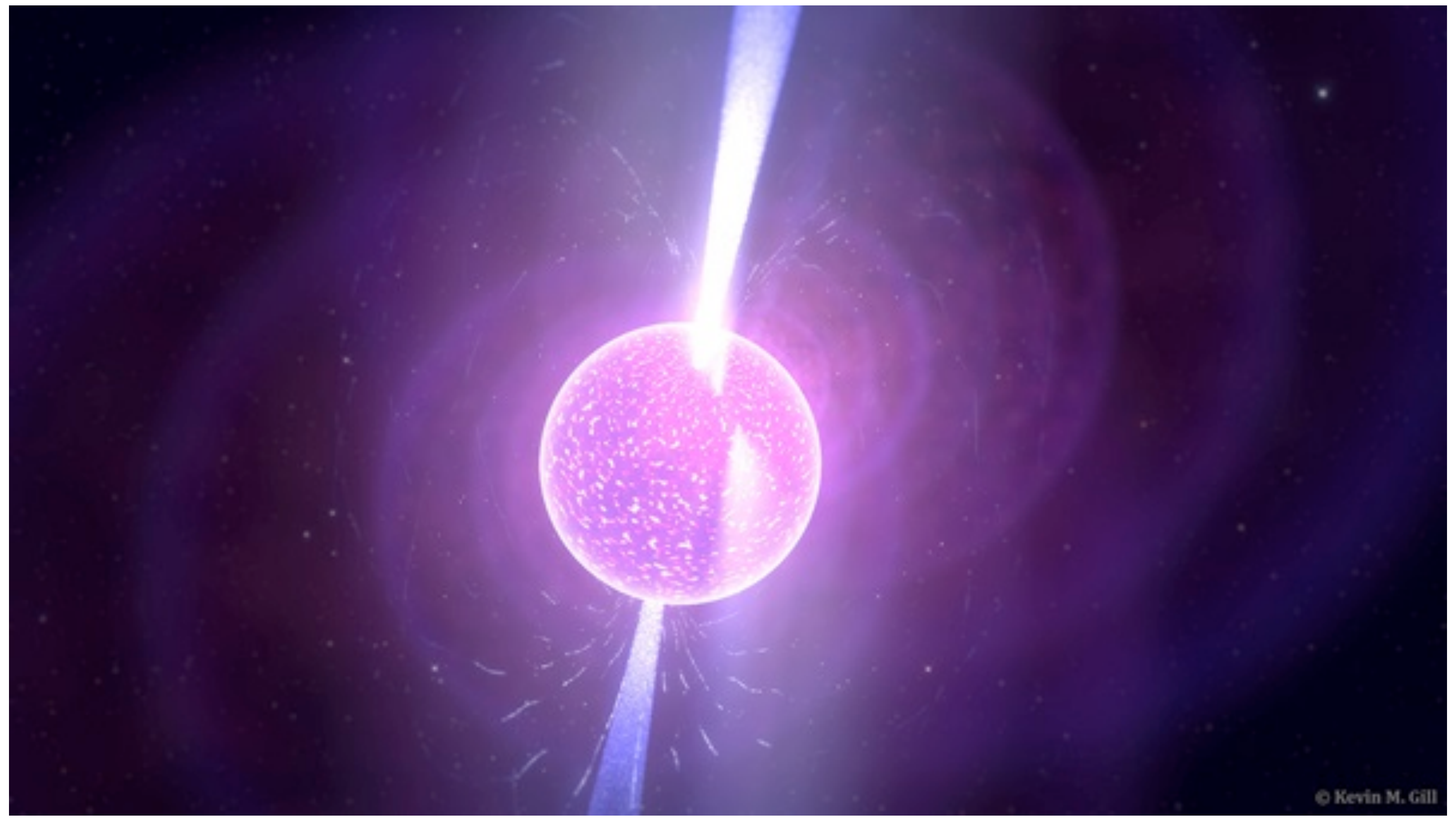
$$L_{\mathrm{GW}} = \frac{1}{5}\left(\sum_{j,k} \dddot{I}_{jk}\dddot{I}_{jk} - \frac{1}{3}\dddot{I}^2\right)$$

- Note: L_{GW} is dimensionless in geometric units but can be converted via the scale factor $L_0 = c^5/G = 3.6 \times 10^{52}\mathrm{W}$.

ASTROPHYSICAL SOURCES OF GRAVITATIONAL WAVES

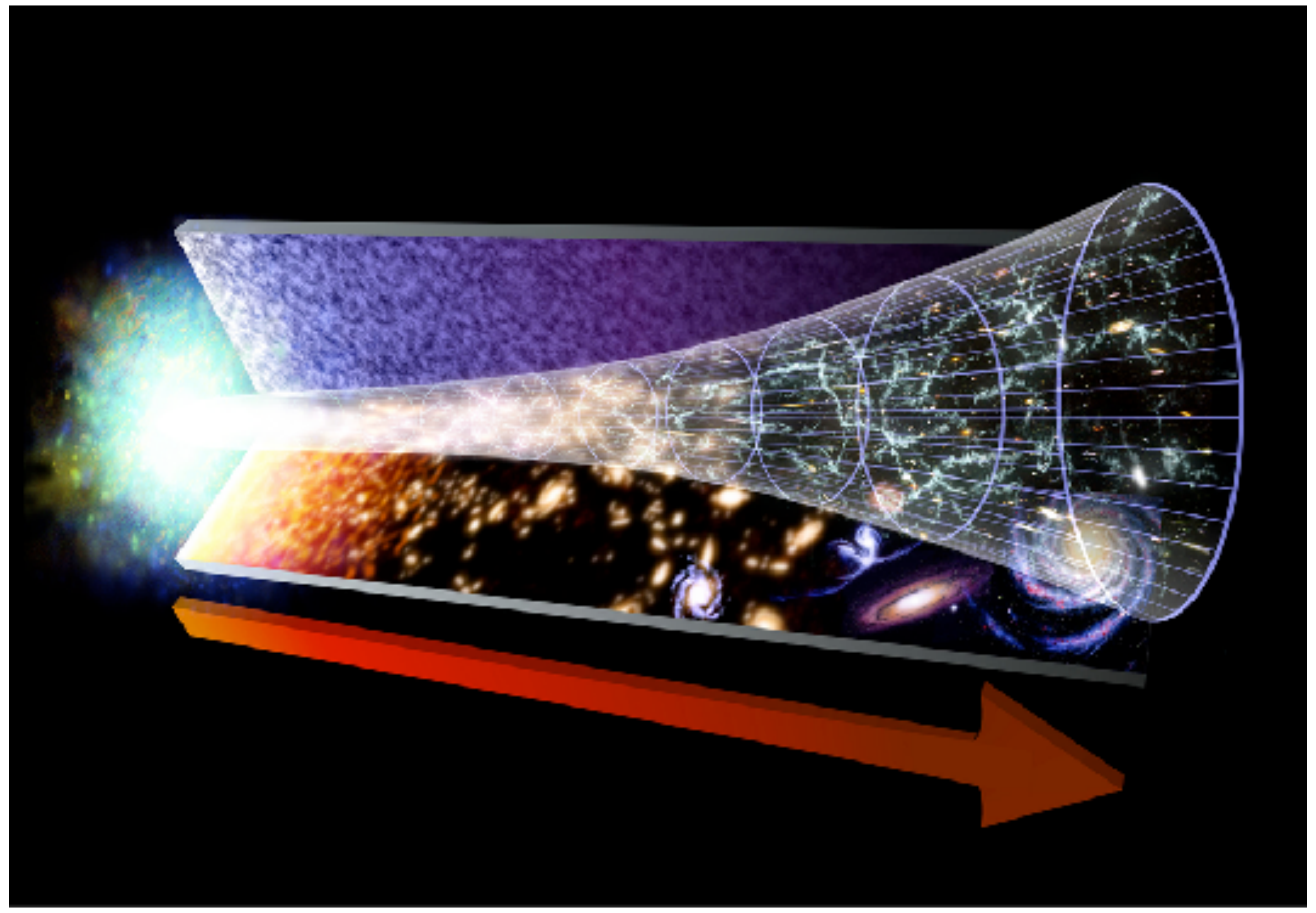
- Any mass distribution with a <u>time-varying quadrupole</u> moment sources gravitational waves

binary black holes,
binary neutron stars



spinning neutron stars,
pulsars, magnetars

supernova explosions



Cosmological stochastic
GW background

COMPACT BINARIES

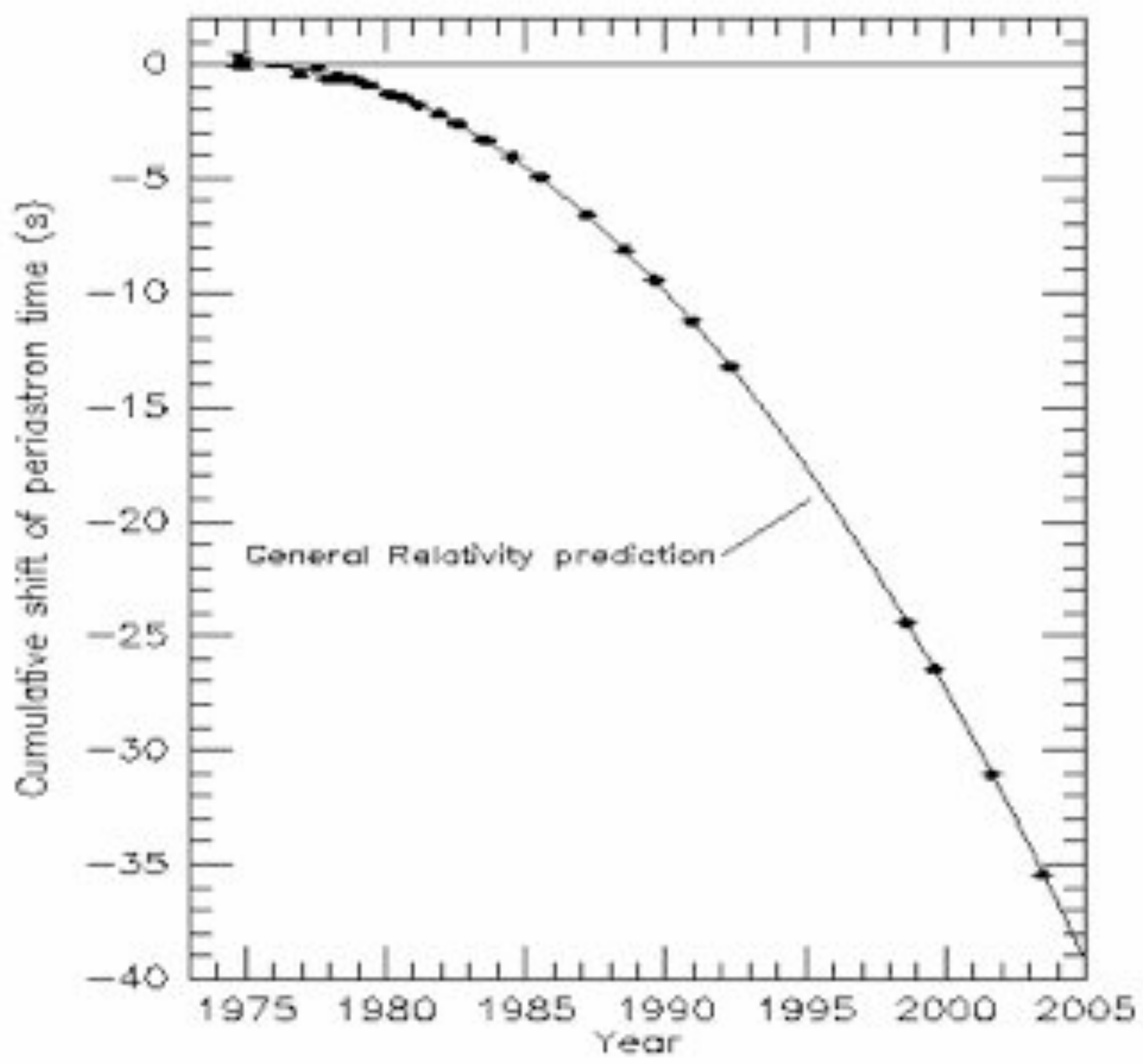
- Binary systems composed of black holes and neutron stars (also white dwarfs, supermassive black holes)
- Their orbital evolution is driven by the emission of gravitational waves, causing the orbit to shrink: **"chirp"** signal
- The GW amplitude of a compact binary can be estimated as

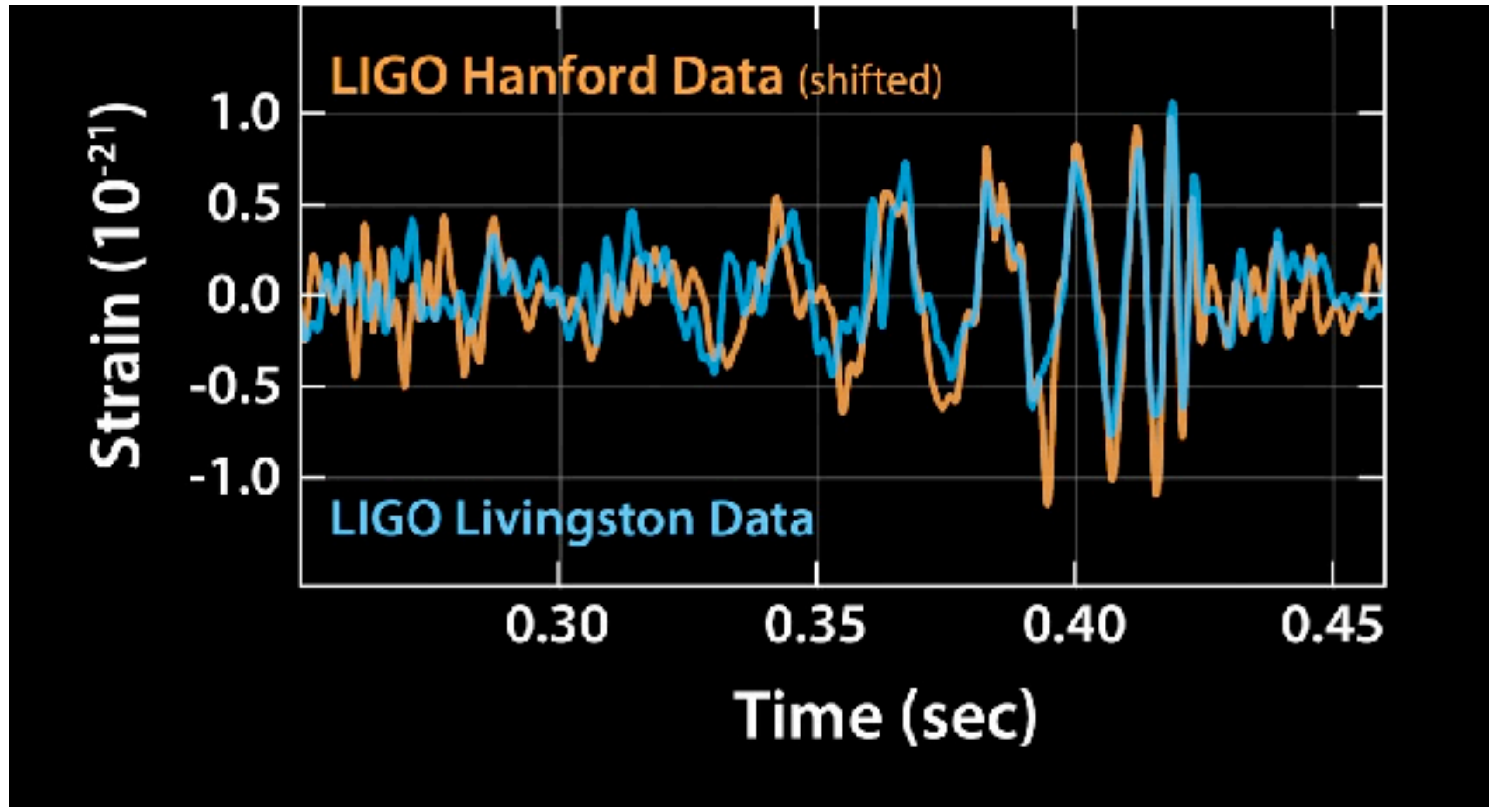
$$h \sim \frac{2}{r}\mathcal{M}_c^{5/3}\omega_{\rm orb}^{2/3}$$

- The characteristic frequency of a compact object can be estimated as

$$f_0 \sim \frac{1}{4\pi}\left(\frac{3M}{R^3}\right)^{1/2} \sim 1\text{kHz}\left(\frac{10M_\odot}{M}\right)$$

- Famous examples: Hulse-Taylor binary pulsar PSR B1913+16, GW150914, GW170817
    - Note: All directly GWs detected to date are consistent with compact binary mergers

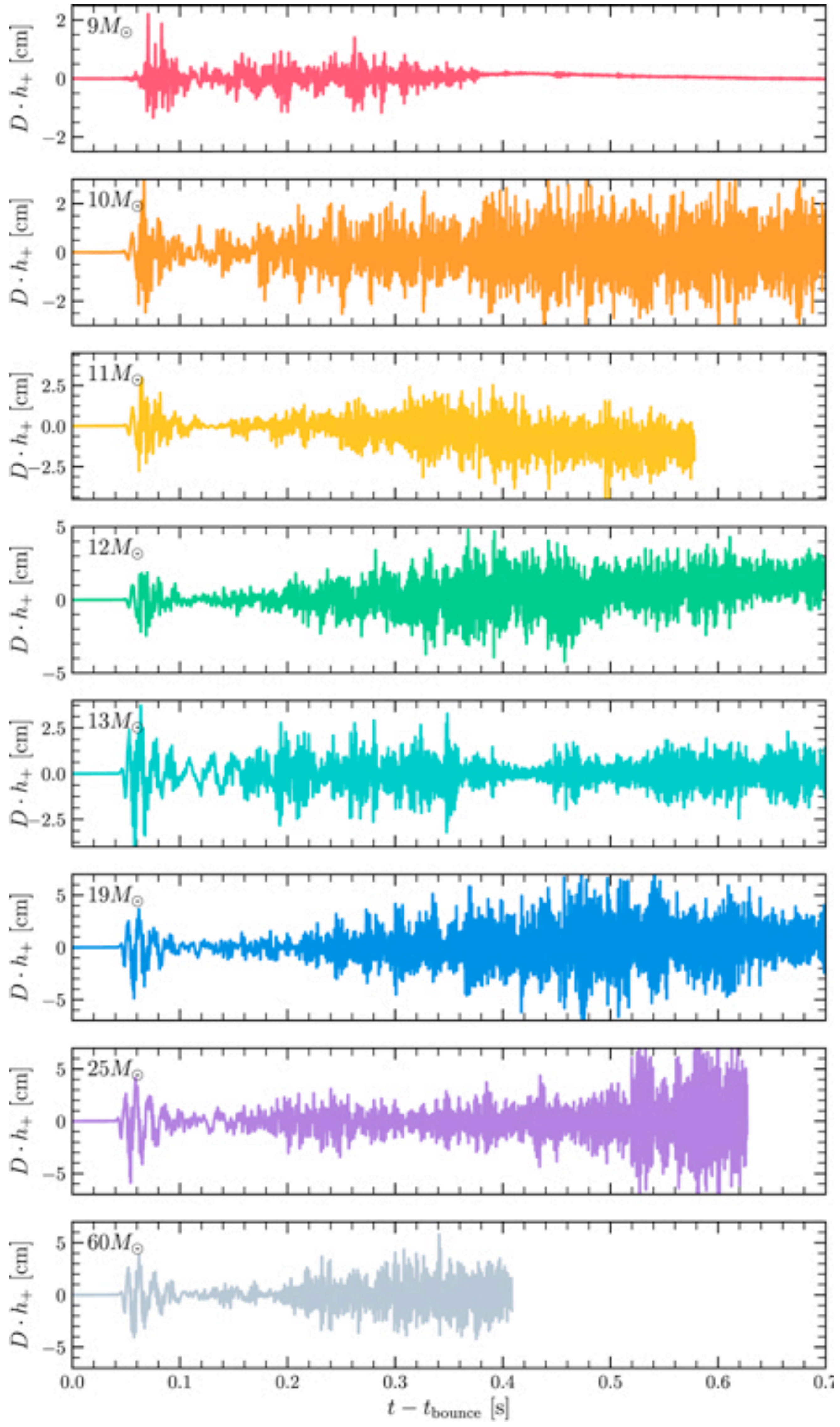




CORE-COLLAPSE SUPERNOVAE

- Type II supernovae: Massive stars ($8M_\odot \lesssim M \lesssim 50M_\odot$) collapse at the end of their life and form either a black hole or a neutron star (remnant)
- If the collapse is non-spherical, GWs can carry away binding energy and angular momentum
- The Type II SNe rate in a Milky Way-like galaxy is 0.01-0.1 per year
- The GW amplitude can be estimated to be

$$h \sim 6 \times 10^{-21} \left(\frac{E_{\rm GW}}{10^{-7} M_\odot}\right)^{1/2} \left(\frac{1\rm ms}{T}\right)^{1/2} \left(\frac{1\rm kHz}{f}\right) \left(\frac{10\rm kpc}{D_L}\right)$$



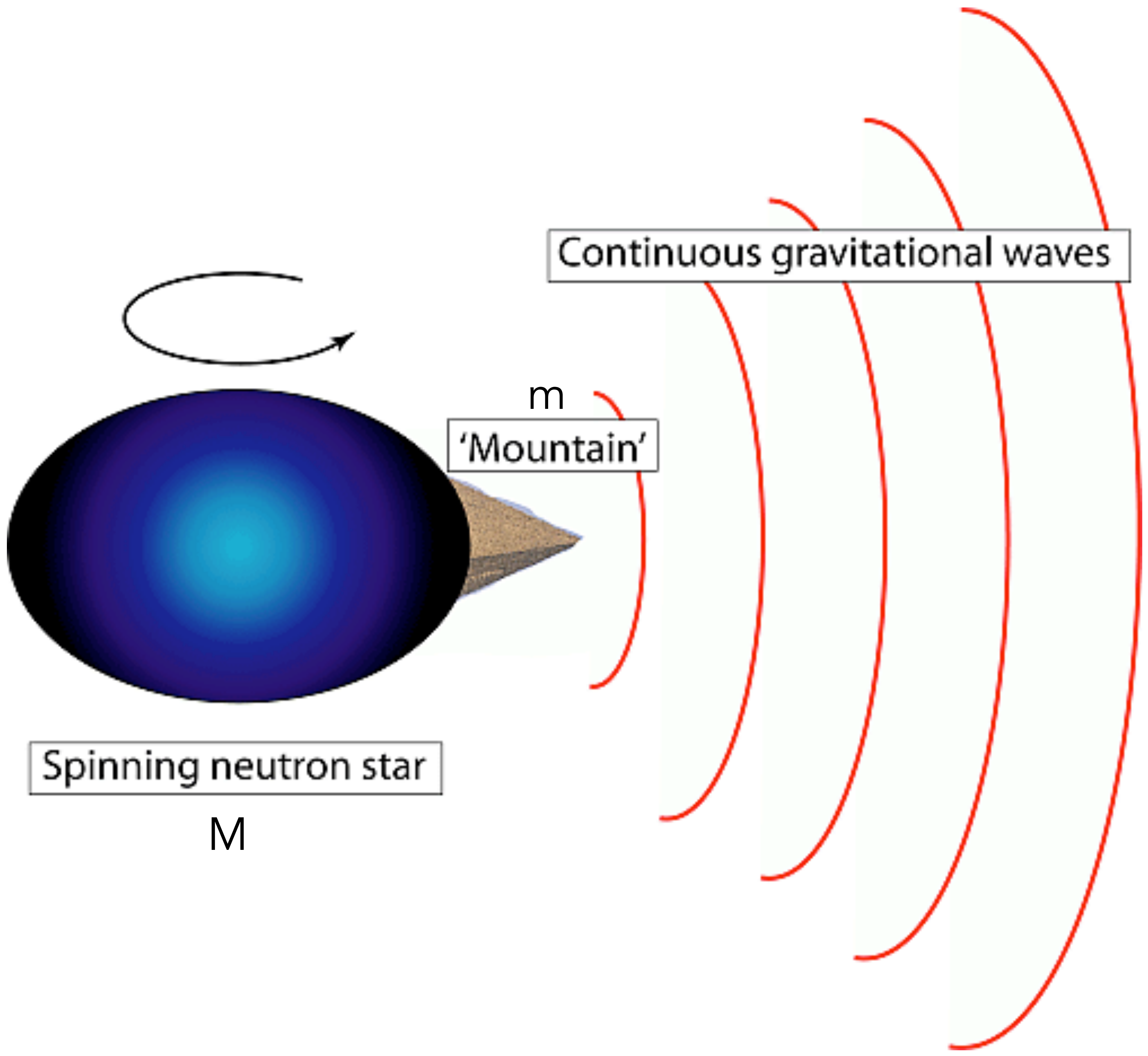
[Radice+, 2019]

ISOLATED NEUTRON STARS

- Gravitational pulsars = rotating neutron stars with asymmetry (" neutron star mountain")
- The asymmetry leads to a non-symmetric quadrupole tensor
    - Assume a star with uniform density. Its moment of inertia is given by $I = 2MR^2/5$. A mountain with mass m will introduce a fractional asymmetry

$$\epsilon = \frac{5m}{2M}$$

    - As the star rotates, the mountain will emit GWs, causing the star to spin-down.
- Note: non-observation allows to set an upper limit on ϵ.

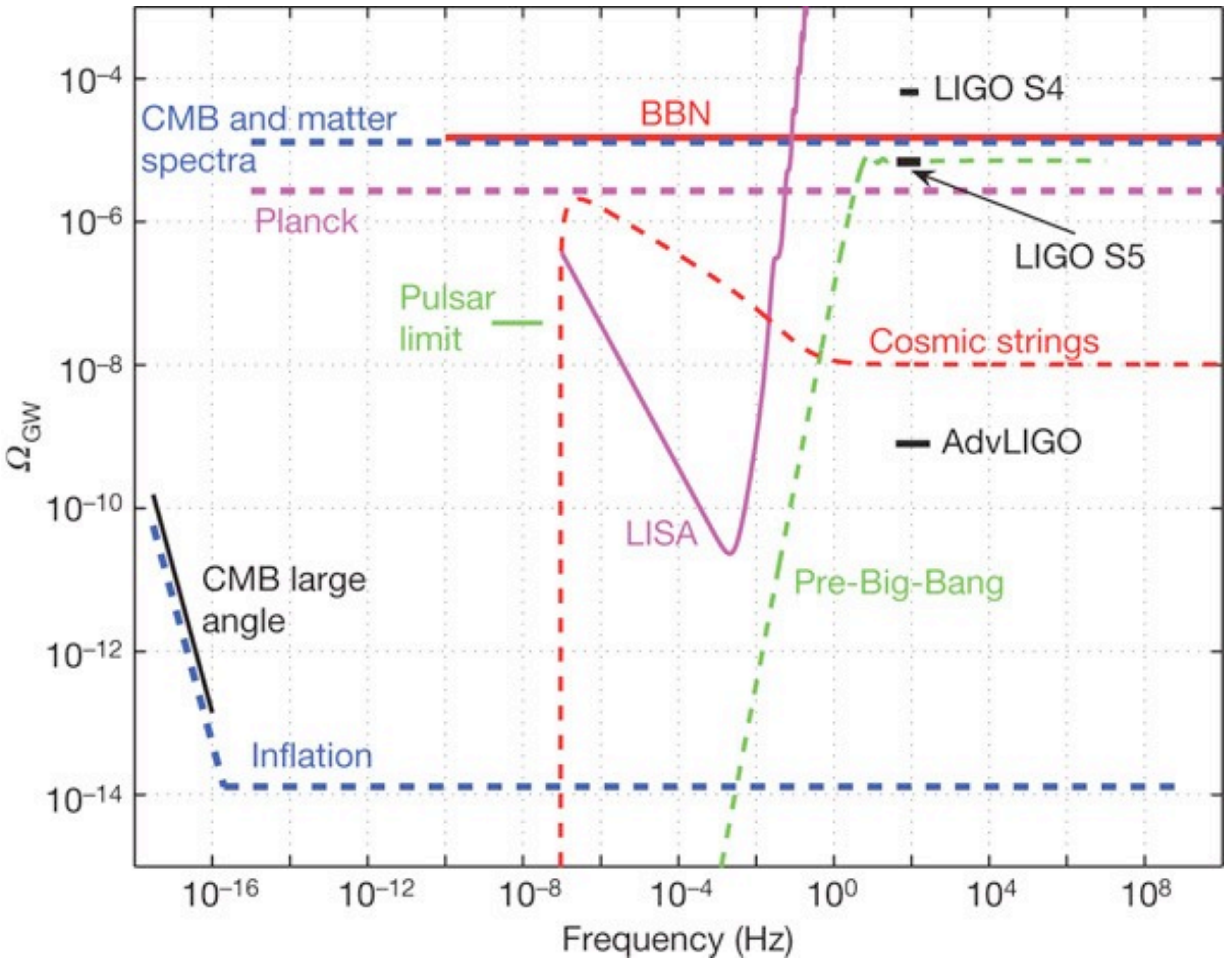


Credit: Astrobites

STOCHASTIC GW BACKGROUND

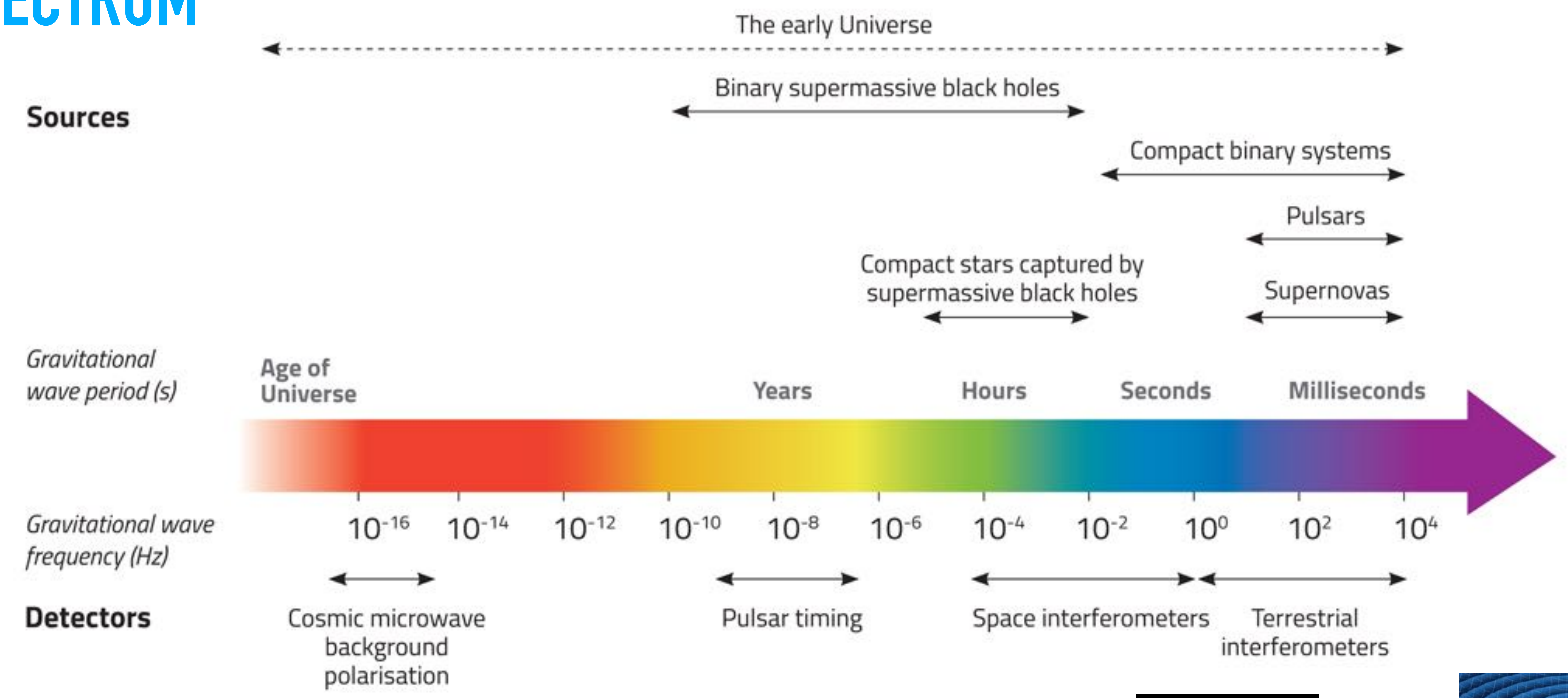
- Superposition of astrophysical events that cannot be resolved individually
- Background from **fundamental processes** in the early universe, e.g. the Big Bang
  - Expected to be very weak but will allow us to look back at the universe when it was $10^{-30}s$ old and at very high energies!
  - Characterised by the energy density of a random field of gravitational waves with a mean square amplitude per unit frequency $S_{gw}(f)$.
  - The SGWB density parameter is then given by:

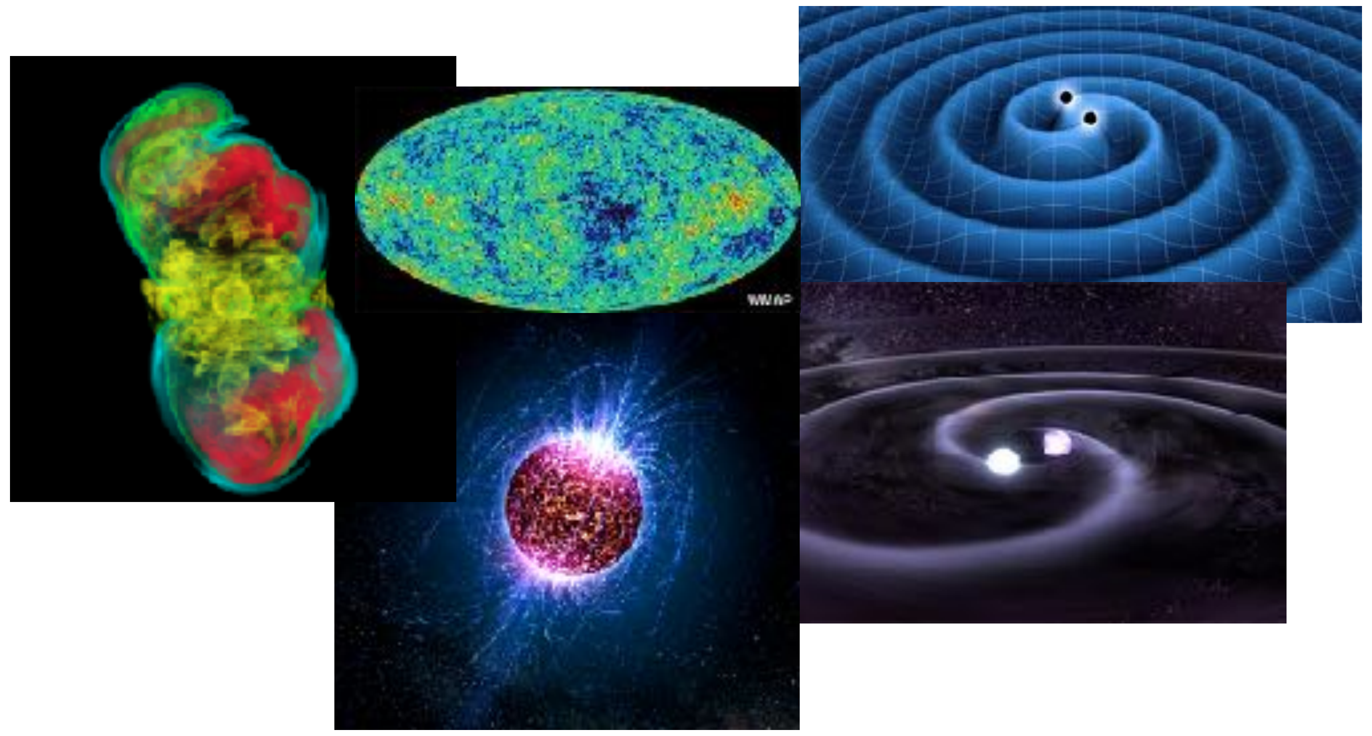
$$\Omega_{gw}(f) = \frac{10\pi^2}{3H_0^2} f^3 S_{gw}(f)$$



[LVC, Nature]

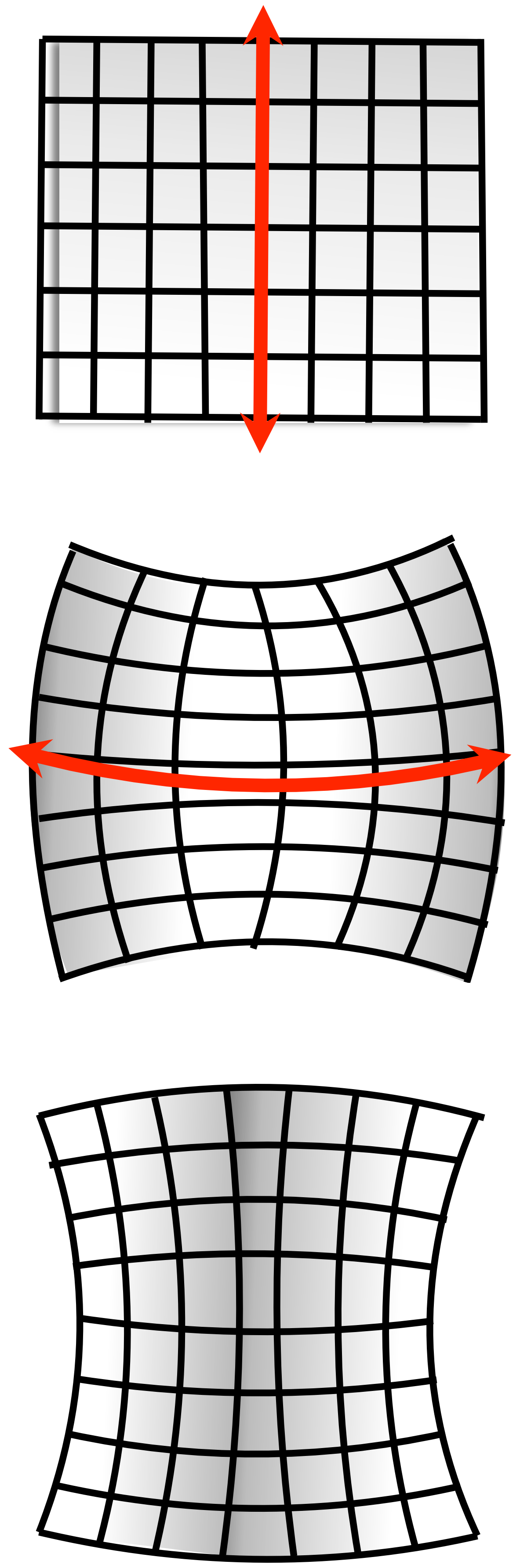
GW SPECTRUM

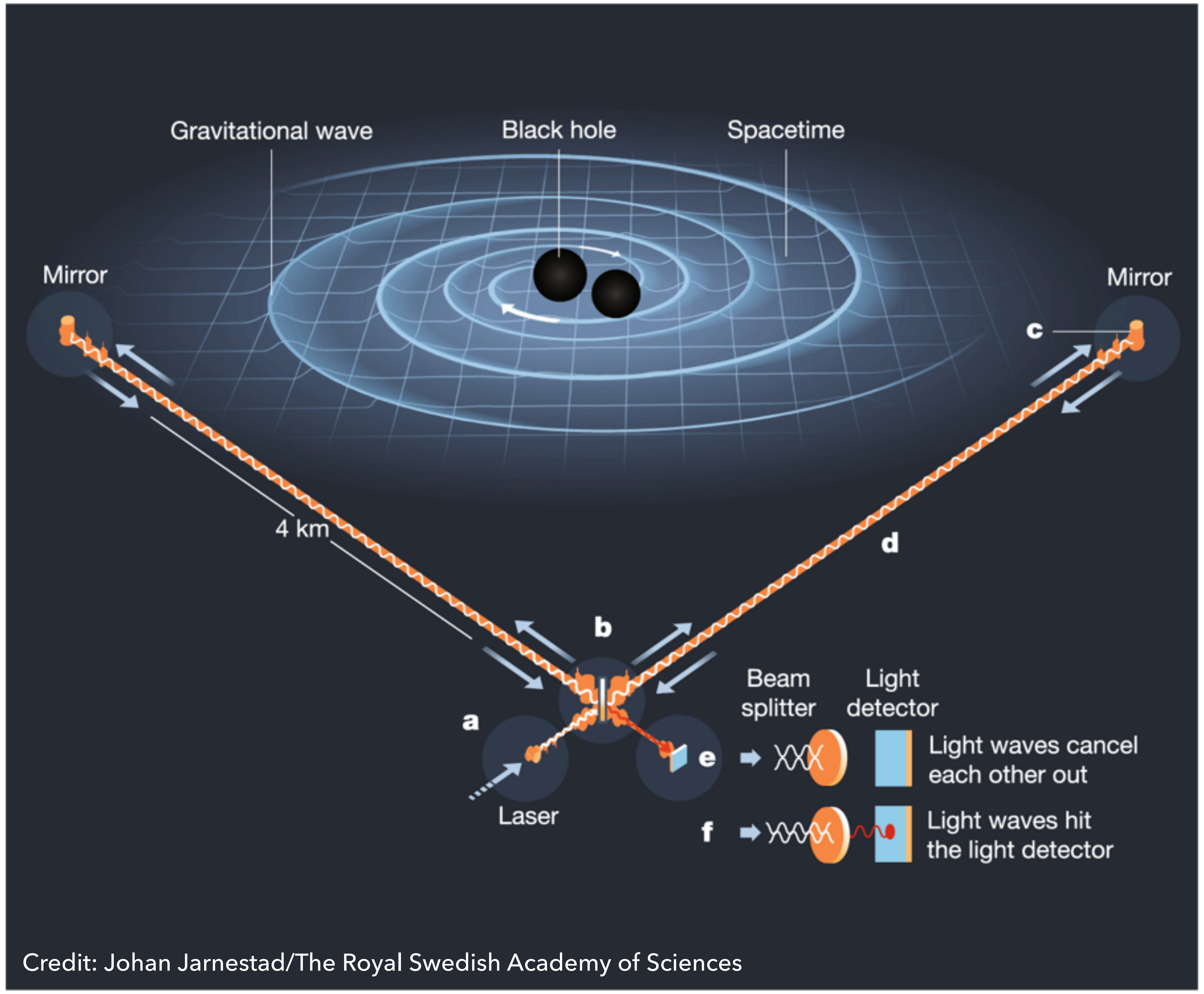

The early Universe
Sources
Binary supermassive black holes
Compact binary systems
Pulsars
Compact stars captured by supermassive black holes
Supernovas
Gravitational wave period (s)
Age of Universe
Years
Hours
Seconds
Milliseconds
Gravitational wave frequency (Hz)
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4}
Detectors
Cosmic microwave background polarisation
Pulsar timing
Space interferometers
Terrestrial interferometers



GRAVITATIONAL-WAVE DETECTORS

- Precision interferometry: Use two (perpendicular) lasers beams to measure the length of each arm



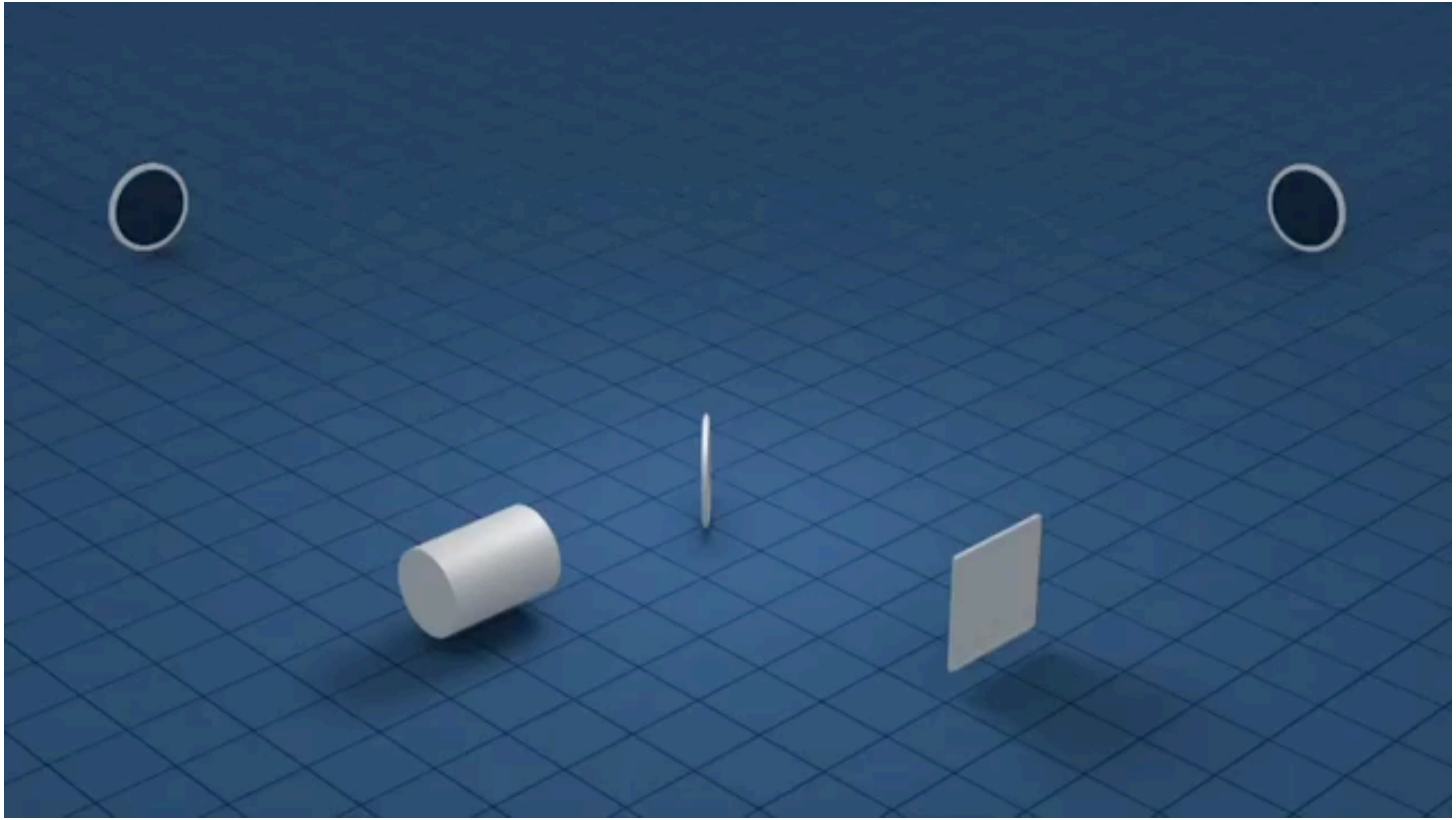


Credit: Johan Jarnestad/The Royal Swedish Academy of Sciences

Fractional change in the length of the arms:

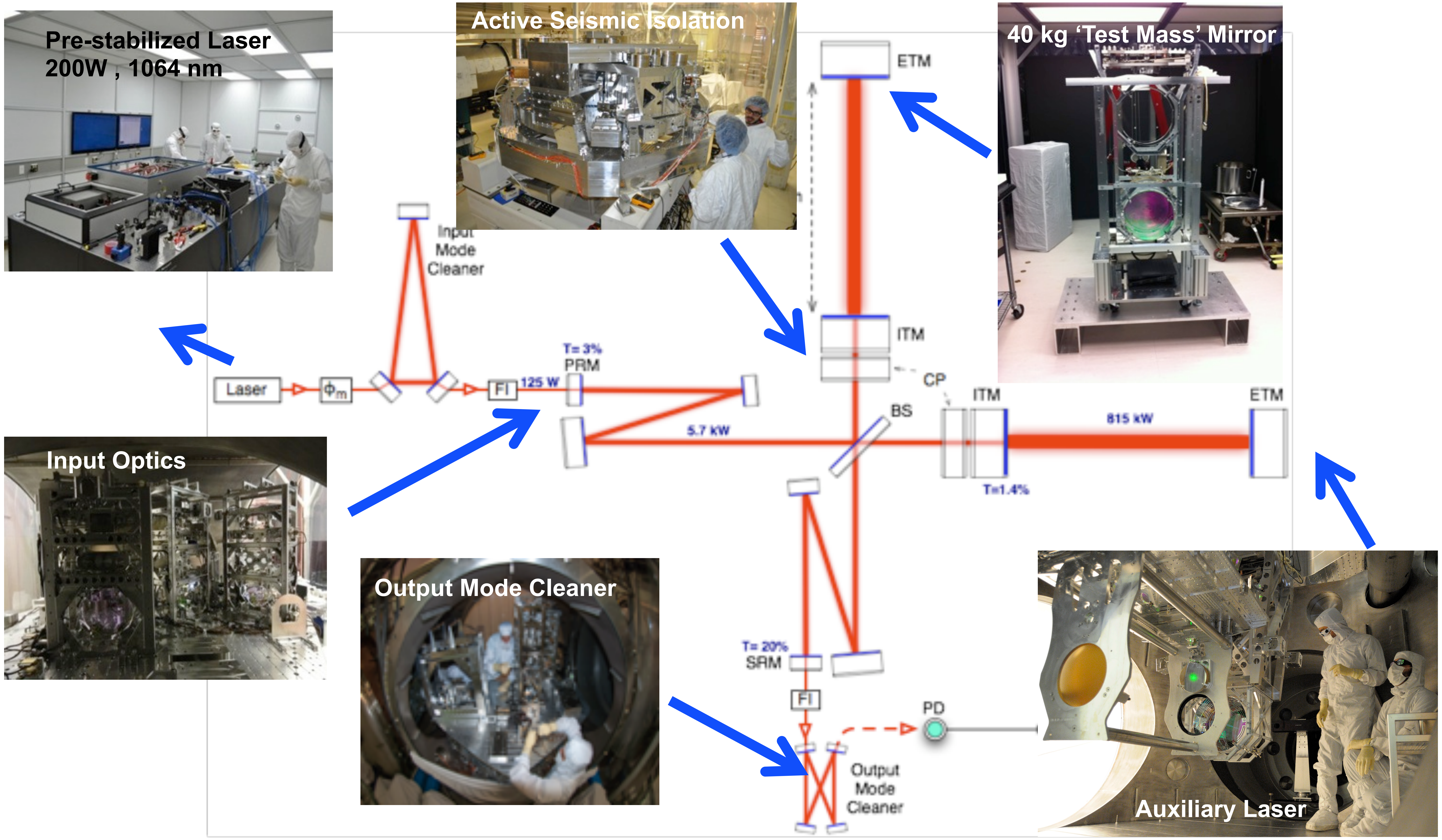
$$\Delta L \sim 10^{-18}\,\mathrm{m}$$

GRAVITATIONAL-WAVE DETECTORS



[Credit:LIGO]

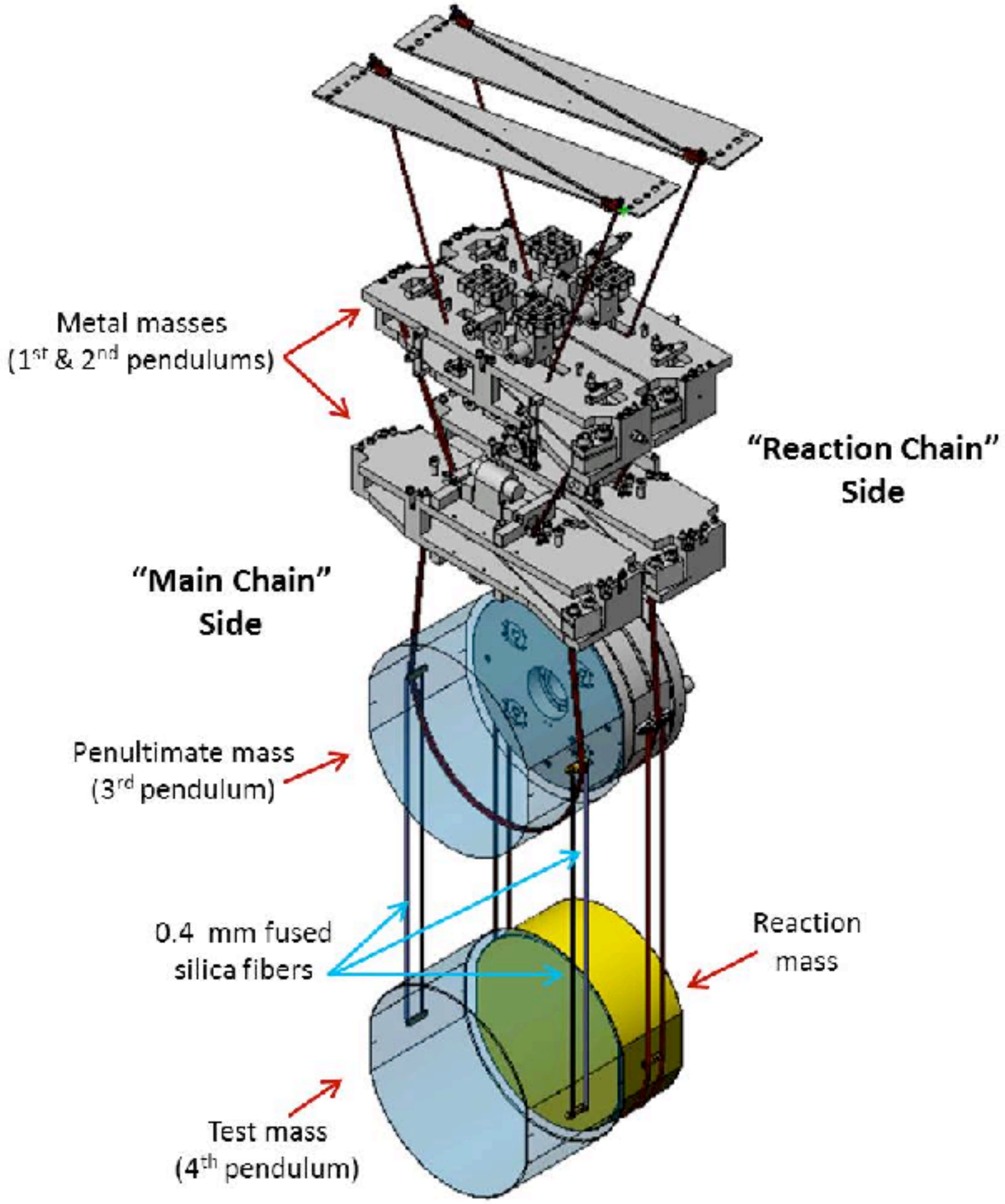
A CLOSER LOOK AT ADVANCED LIGO



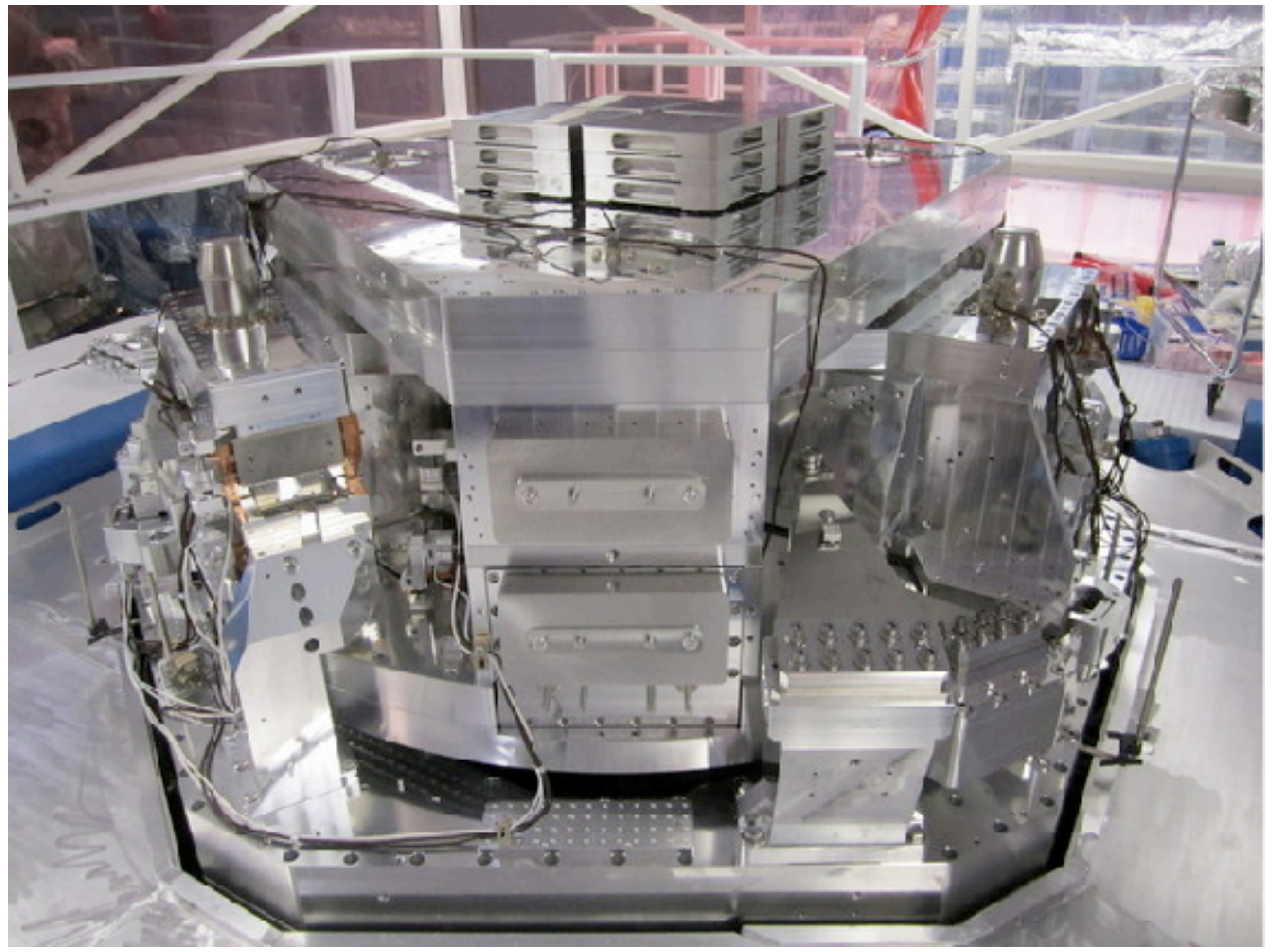
Credit: D. Reitze

A CLOSER LOOK AT ADVANCED LIGO

Multi-stage suspension system to reduce seismic noise

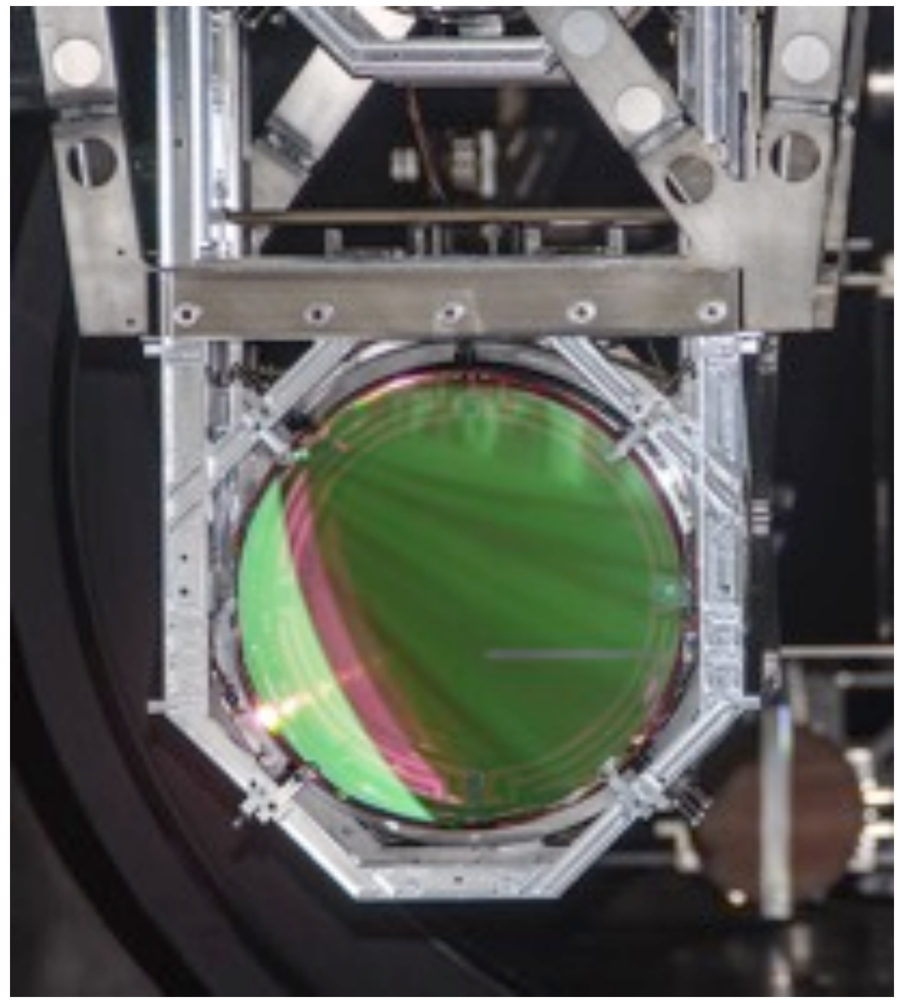


Active seismic damping platform



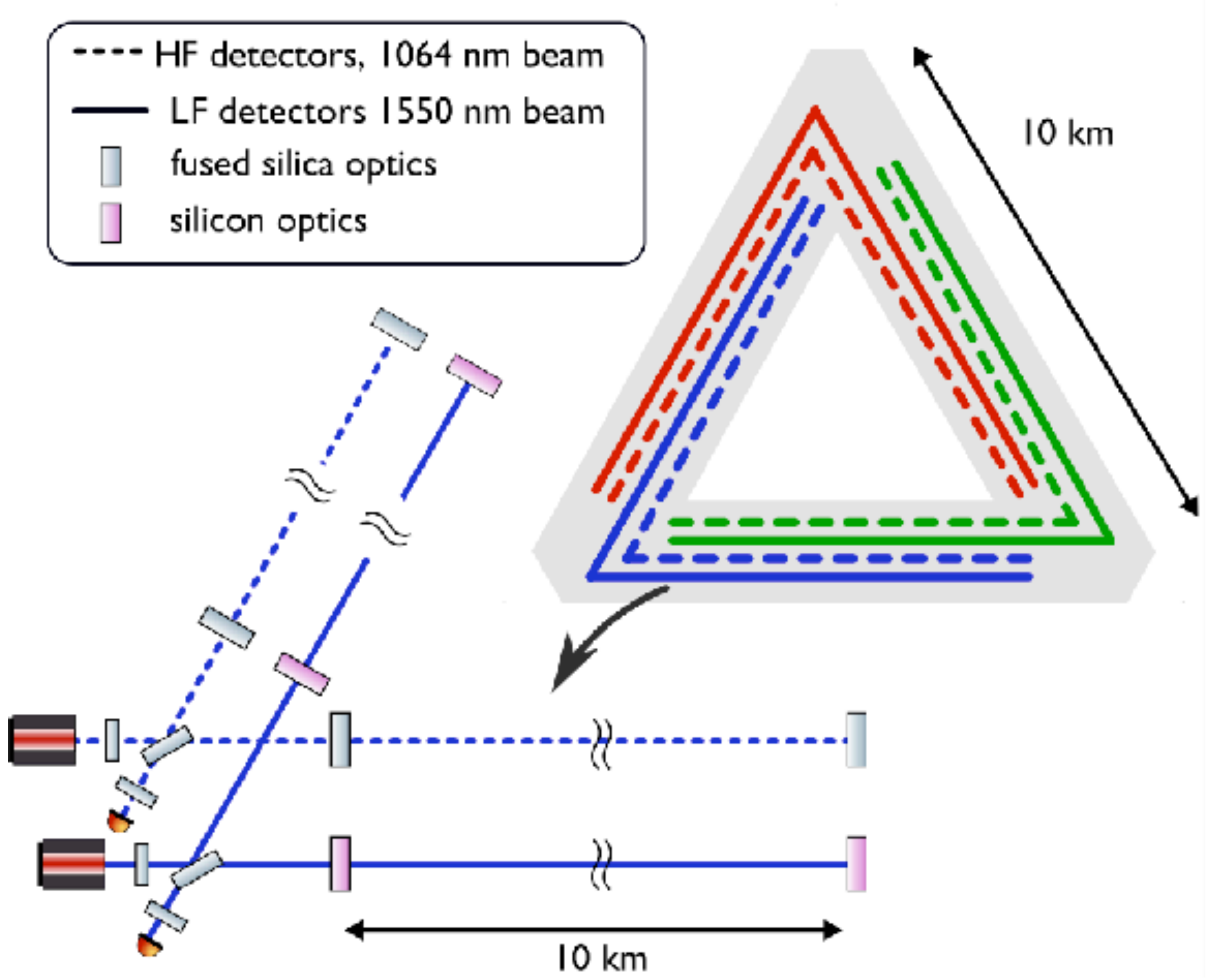
A CLOSER LOOK AT ADVANCED LIGO

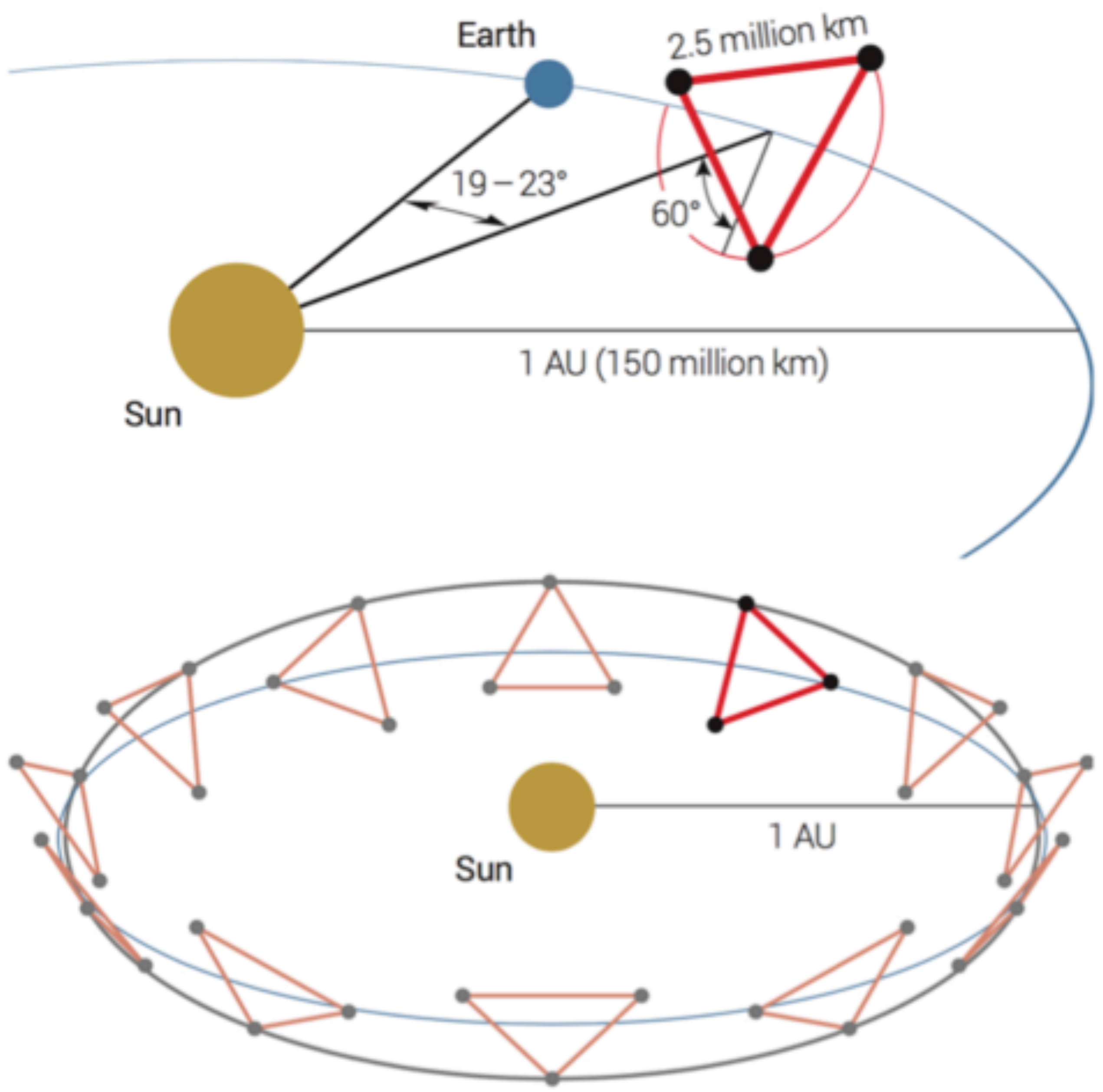
- Vacuum system for ultra-pure vacuum
  - Volume: ~9000m^3
  - Atmospheric pressure inside the tubes ~ 10^{-8}-10^{-9} Torr
  - Air molecules transfer heat onto mirrors and mimic GWs; dust can damage the mirrors
- Pre-stabilised laser + amplification
  - Input laser power in O3: 70W
  - Laser power is crucial to increase the resolution
- Mirrors: pure fused silica glass at 40kg each
  - 34 x 20 cm
  - 1-in-3-million photons get absorbed
  - Mirrors refocus the laser

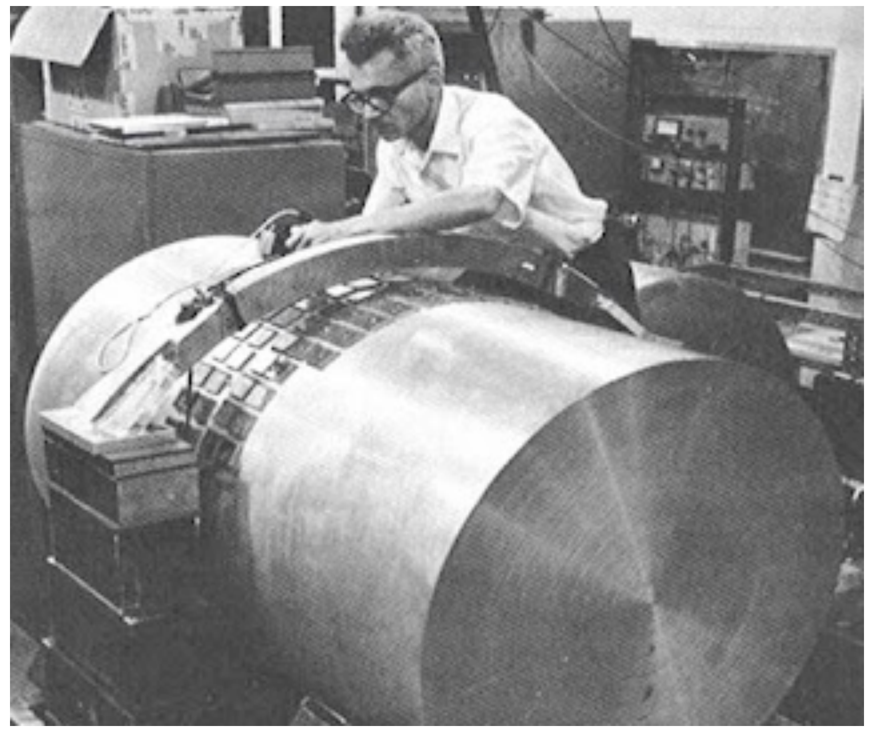


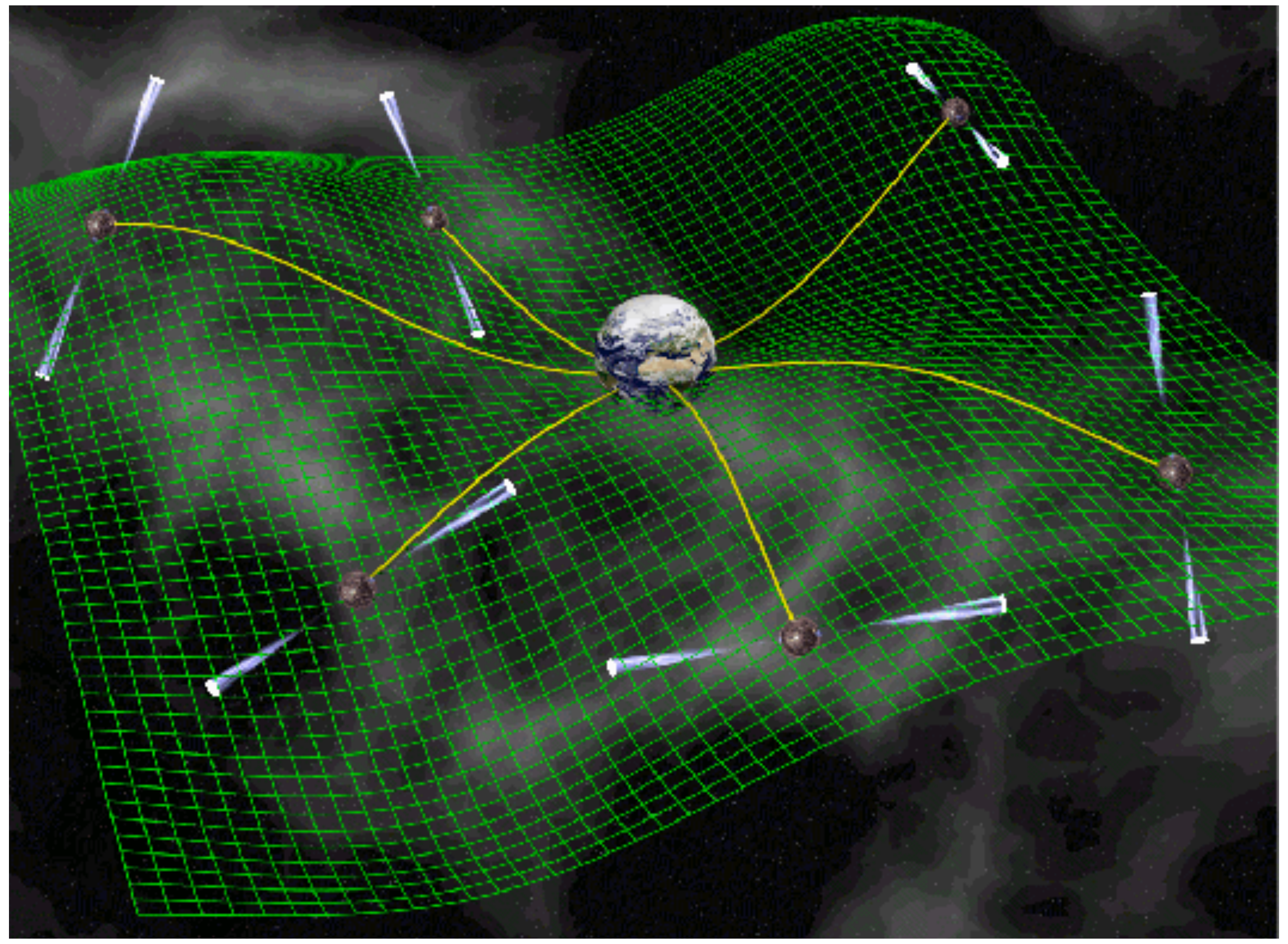
OTHER DETECTOR CONFIGURATIONS

- Triangular interferometers, e.g.
  - Einstein Telescope: proposed third generation ground-based detector
  - LISA: planned space-based mission
- Resonant bar detectors
- Pulsar timing arrays









SENSITIVITY

- The sensitivity of GW detector is characterised by the **power spectral density (PSD)** of its noise background in the absence of a GW signal.

- Data is recorded as a time series: $(n(t_0), n(t_1), \ldots, n(t_n))$

    - Discrete Fourier transform: $(\tilde{n}(f_0), \tilde{n}(f_1), \ldots, \tilde{n}(f_n))$

- Let us assume that the noise is **stationary and Gaussian**. Then the probability density of one realisation of noise per frequency bin is given by $p(\tilde{n}(f_i)) \propto e^{-|\tilde{n}(f_i)|^2/(2\sigma_i^2)}$ and total probability density for a noise realisation is $p(n) = \prod_{i=0}^{n} p(\tilde{n}(f_i))$.

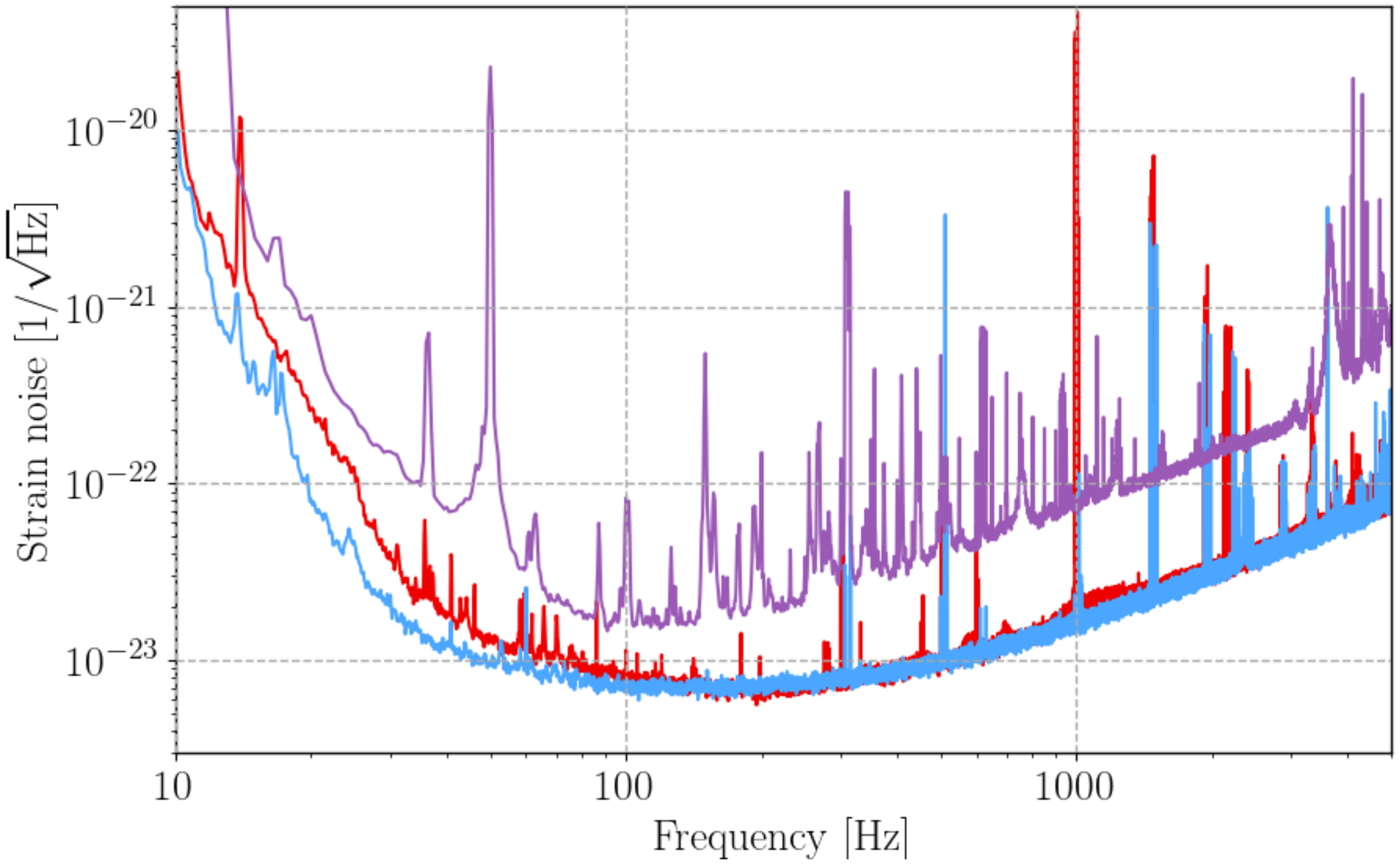
    - In the continuum limit: $p(n) = \mathcal{N} e^{-\frac{1}{2}\sum_{i=1}^{n} \frac{|\tilde{n}(f_i)|^2}{\sigma_i^2}} \rightarrow \mathcal{N} e^{-\int_{-\infty}^{\infty} \frac{|\tilde{n}(f)|^2}{S_n(f)} df}$

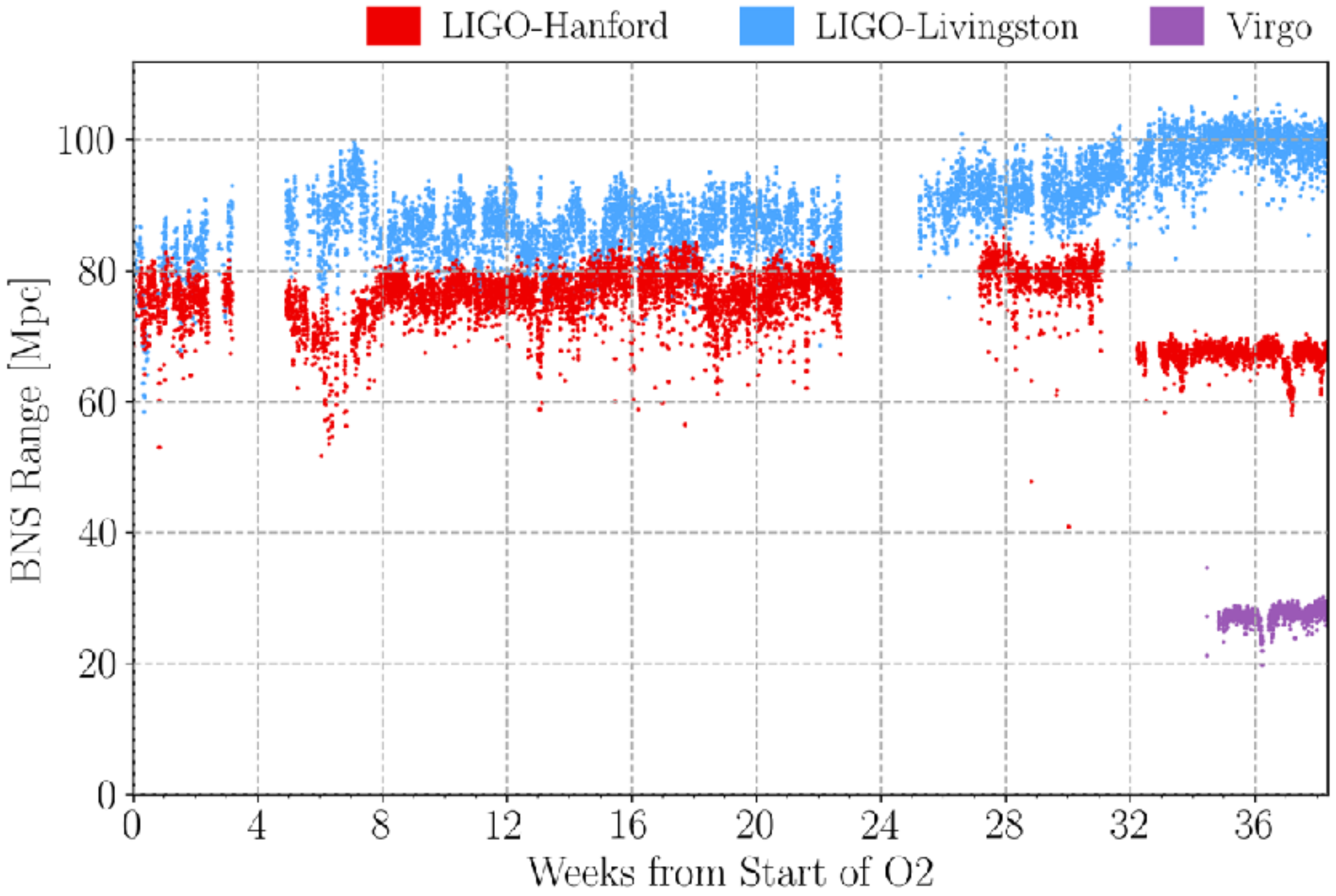
    - $S_n(f)$ is the **noise power spectral density** - the Fourier transform of the noise autocorrelation function:

$$\langle \tilde{n}(f)\tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f')$$

SENSITIVITY DURING O1/O2

Amplitude spectral density = $\sqrt{\mathrm{PSD}}$



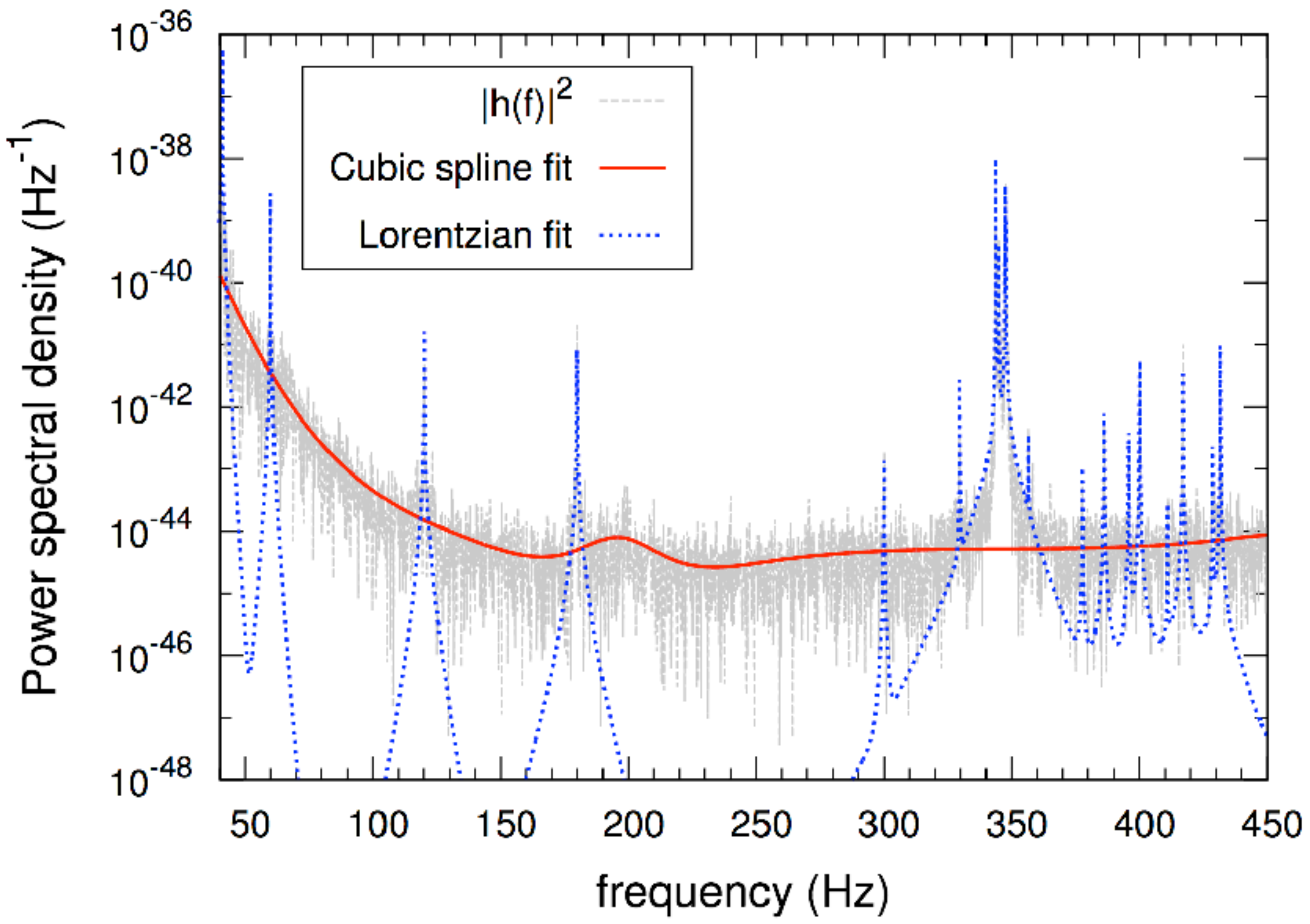


Range: Sky and orientation averaged distance such that a BNS has a SNR of 8

[LVC, GWTC-1]

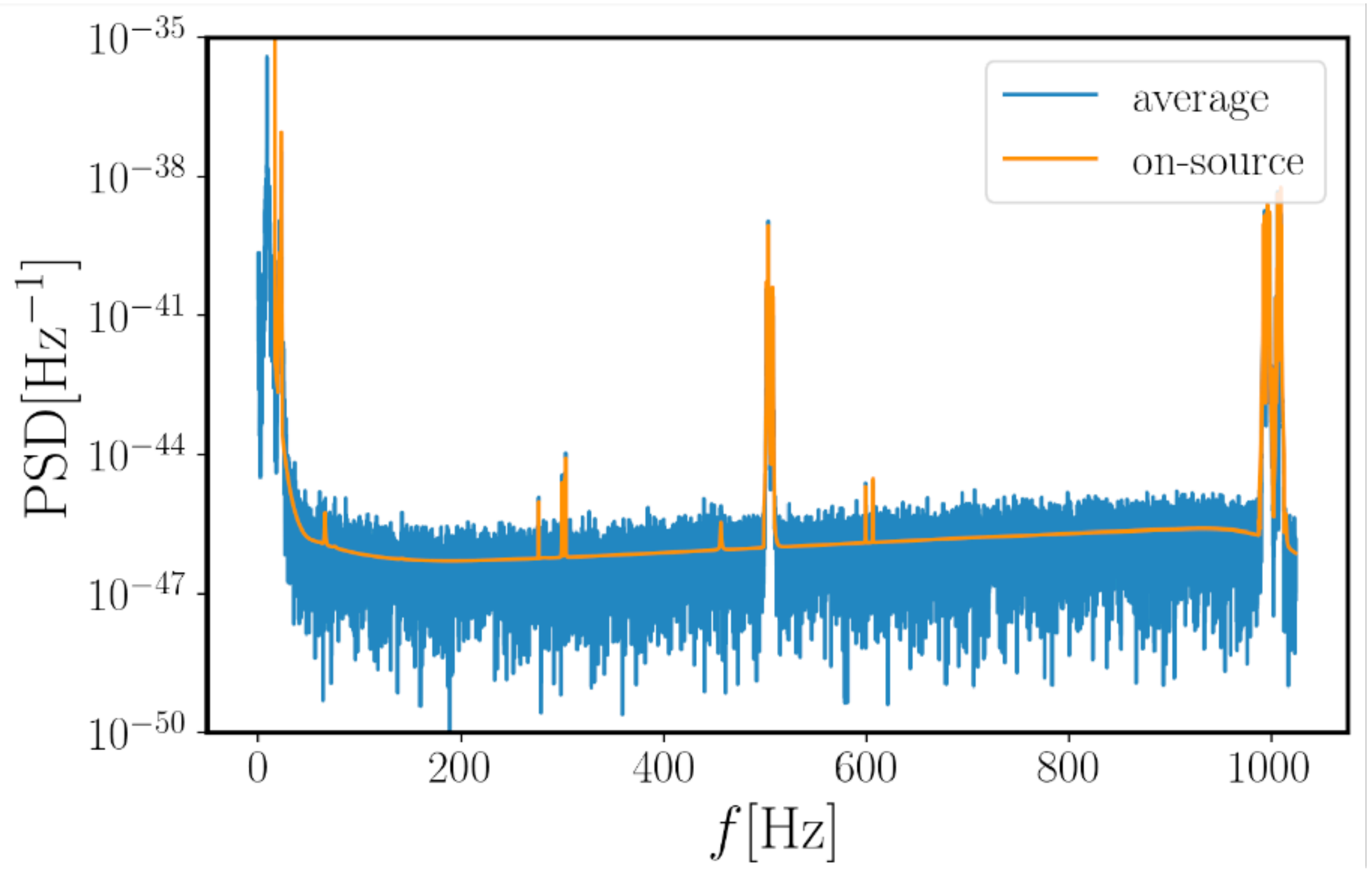
PSD ESTIMATION

- Off-source (average) vs. on-source

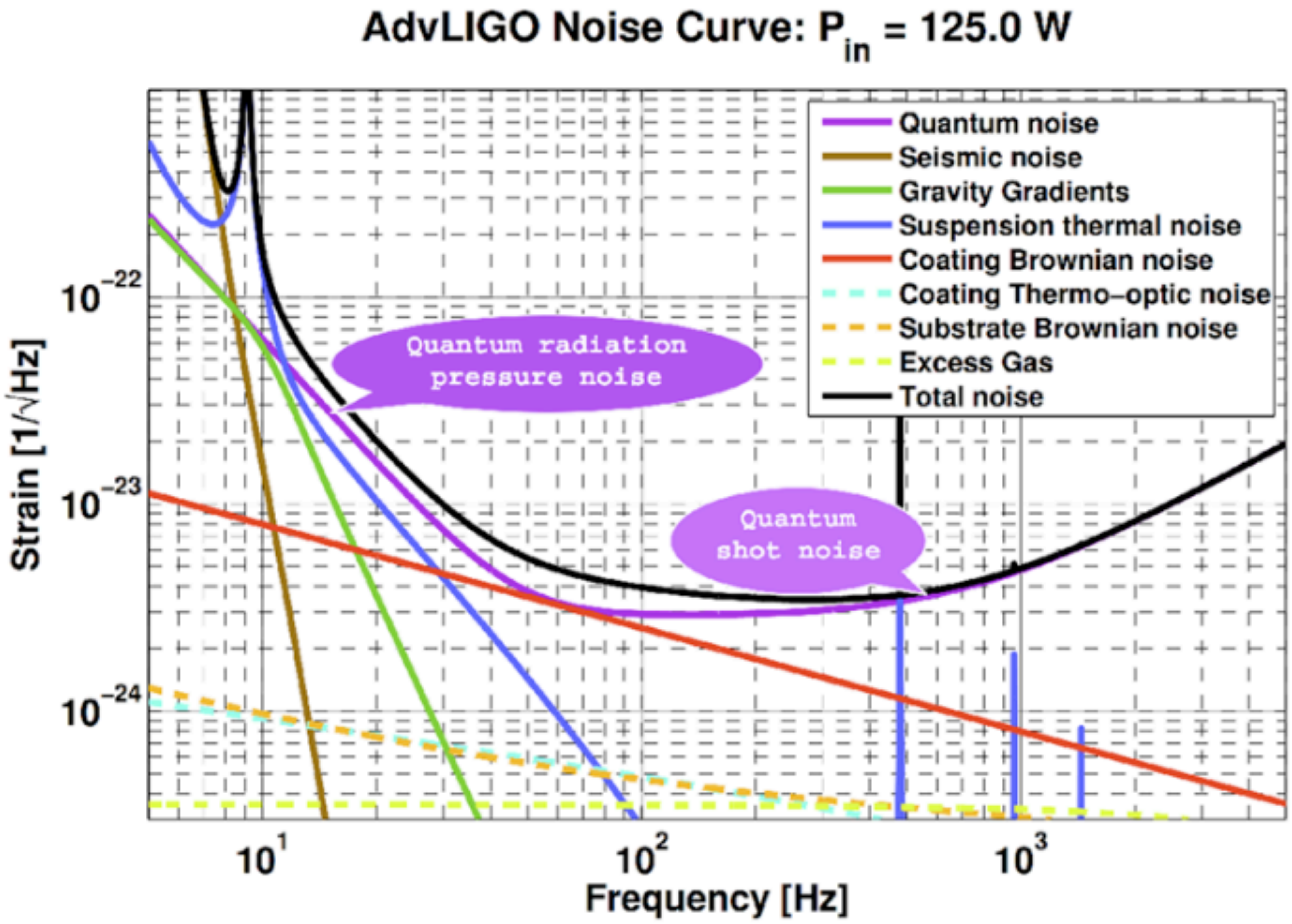


[Littenberg&Cornish, 2014]

Example: GW170608

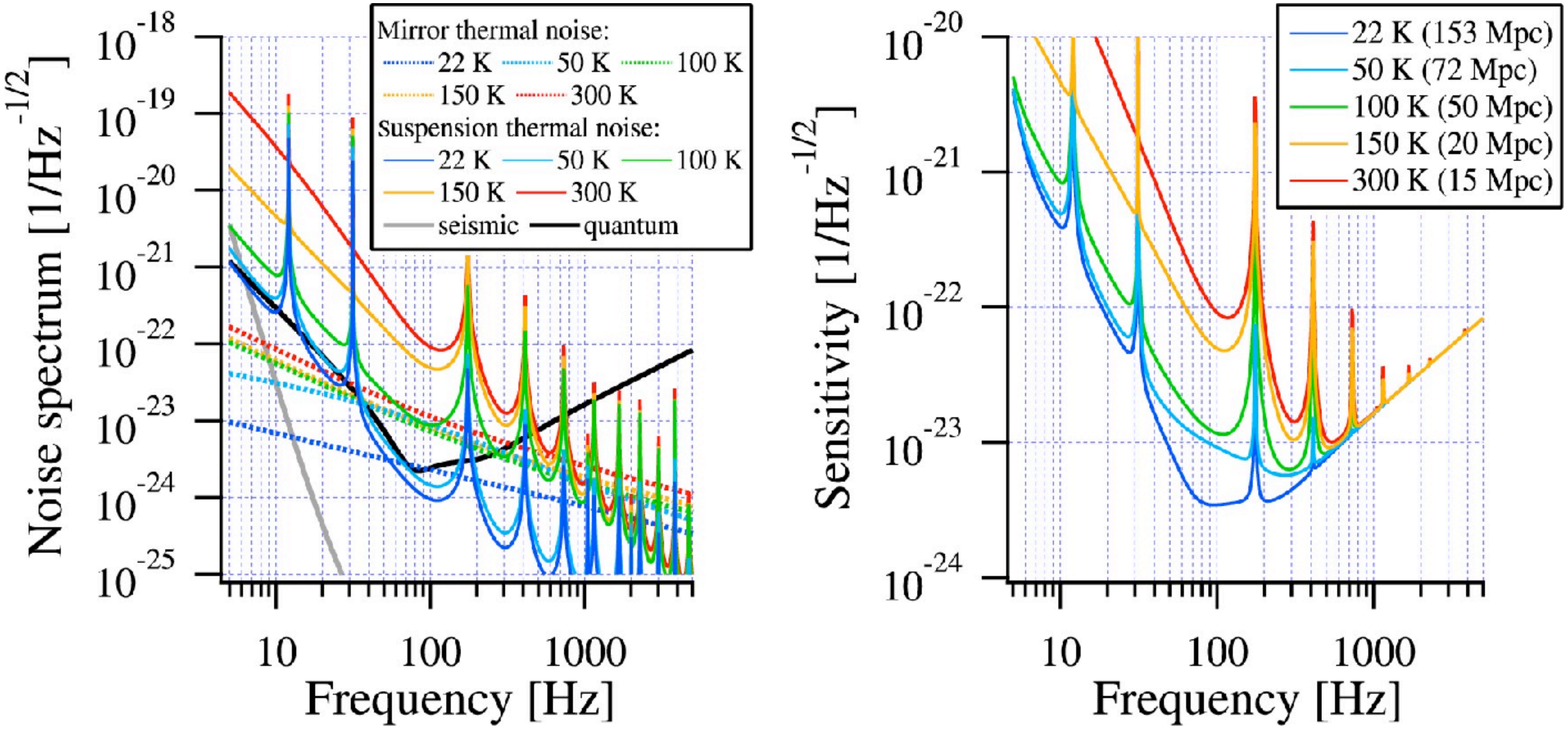


MAJOR SOURCES OF NOISE



[Living Rev. Rel. 2019 22:2]

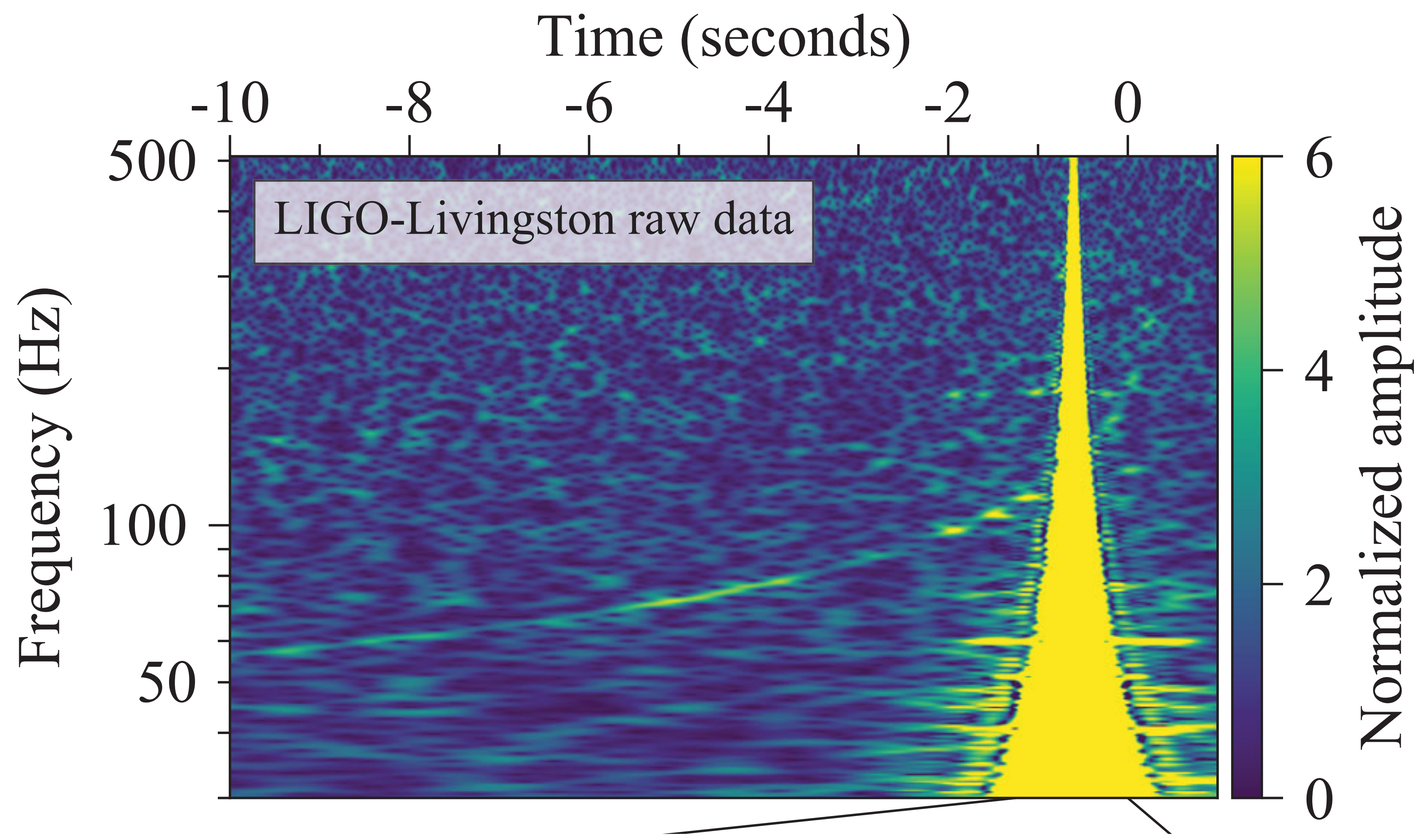
CRYOGENIC COOLING



[KAGRA, 2020]

STATIONARY & GAUSSIAN?

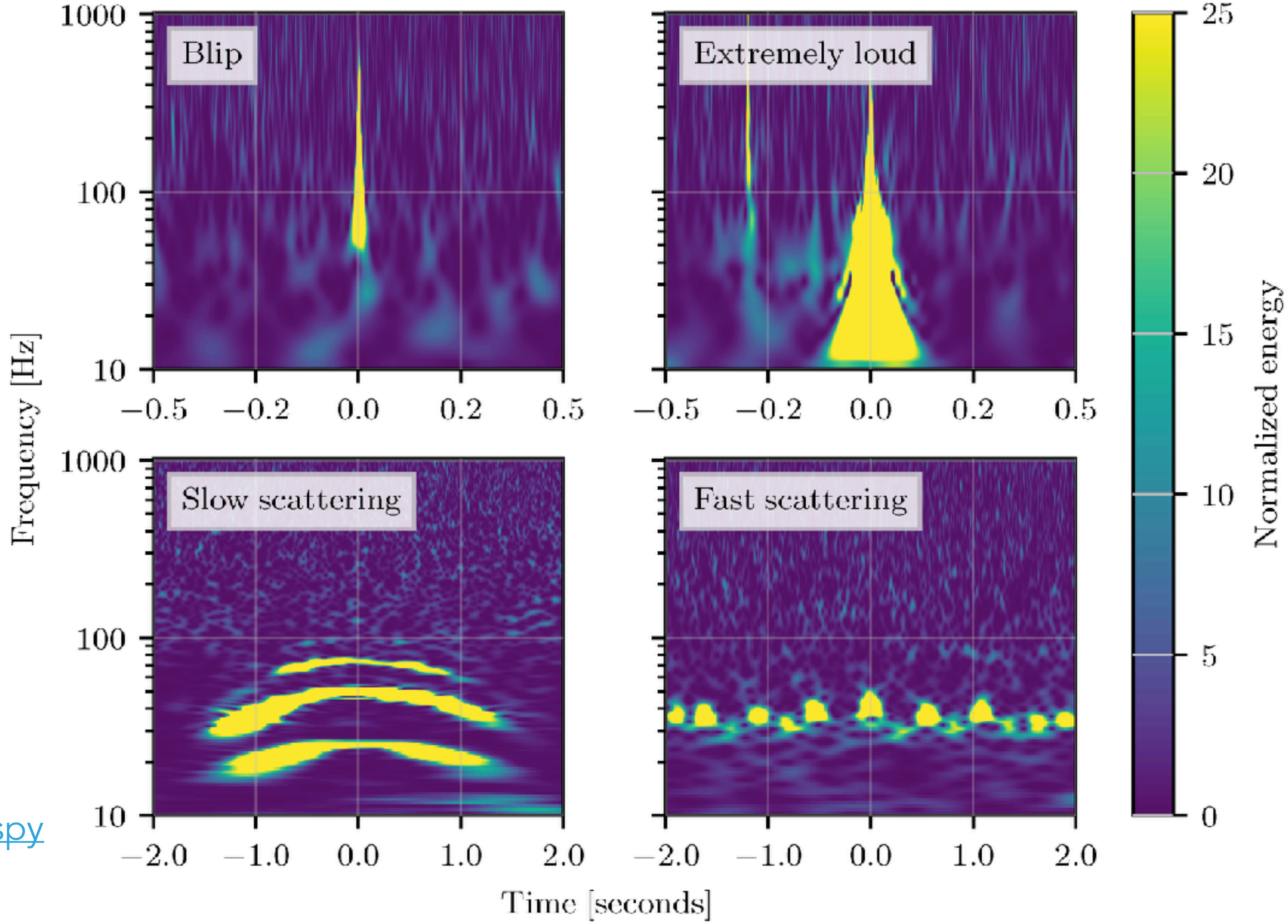
- Data cleaning
- Glitch removal/mitigation



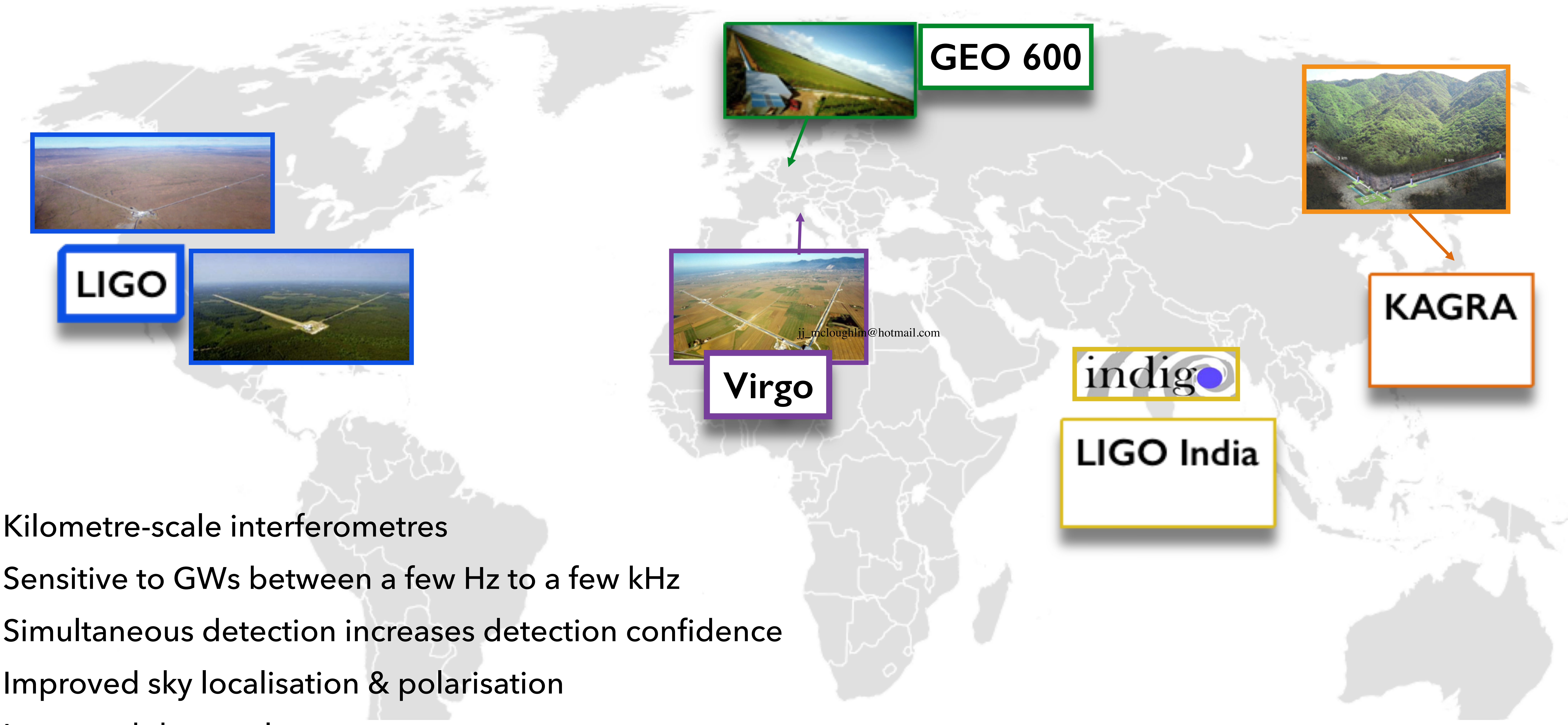
Citizen science: Gravity Spy

https://www.zooniverse.org/projects/zooniverse/gravity-spy

Some example **"glitches"**



DETECTOR NETWORK

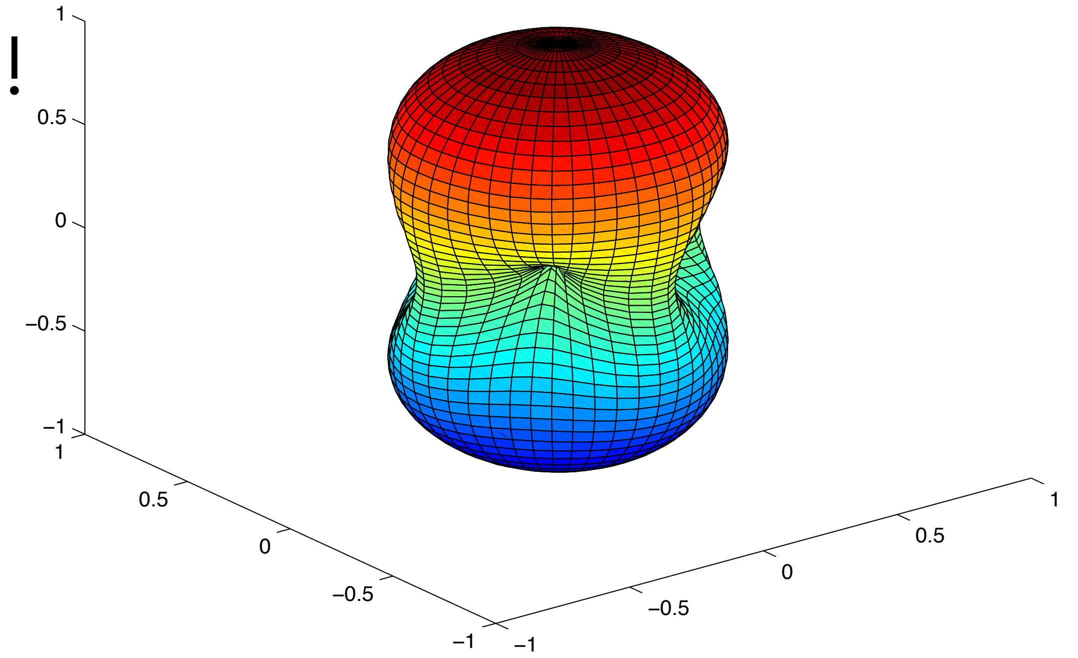


- Kilometre-scale interferometres
- Sensitive to GWs between a few Hz to a few kHz
- Simultaneous detection increases detection confidence
- Improved sky localisation & polarisation
- Increased duty cycle

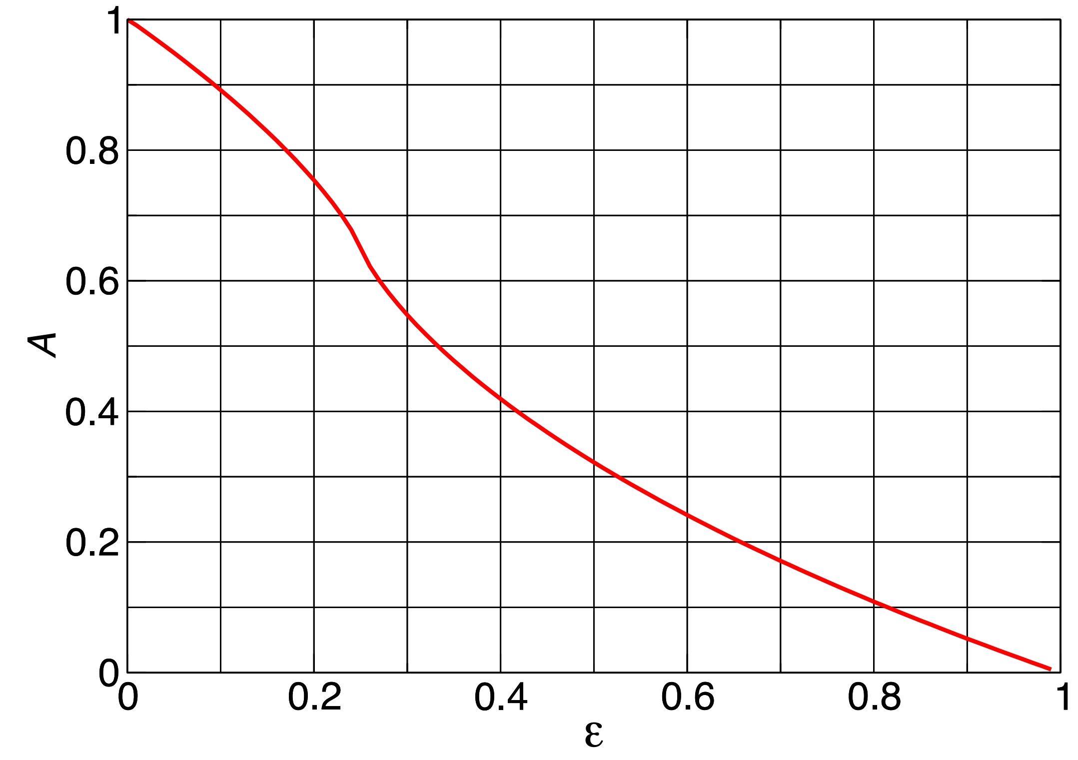
LOCALISATION CAPABILITIES

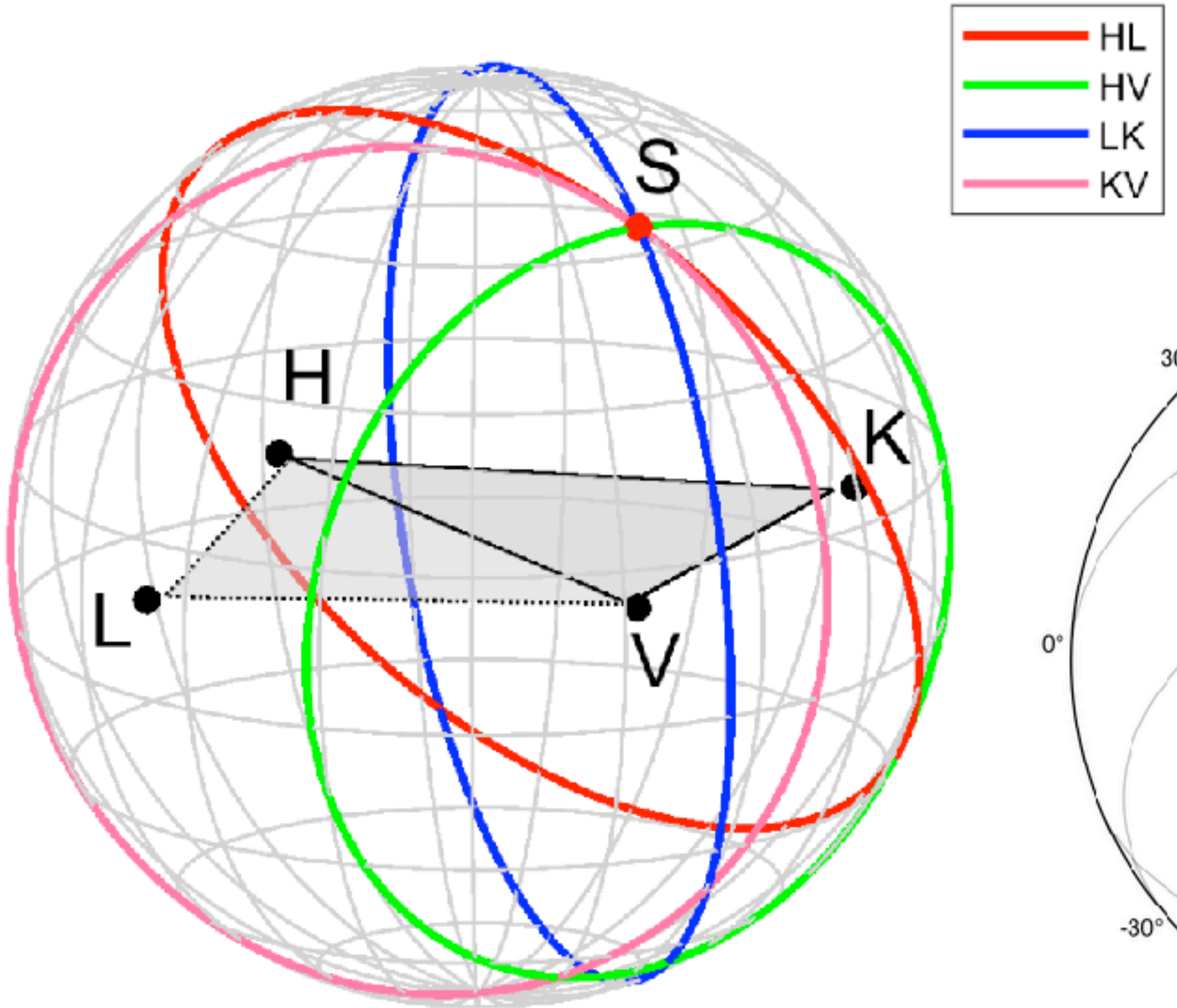
- Individual GW detectors are omnidirectional: poor localisation! Sensitivity depends on location, polarisation and frequency
- Simultaneously operating observatories allow for triangulation via arrival time differences

RMS antenna response

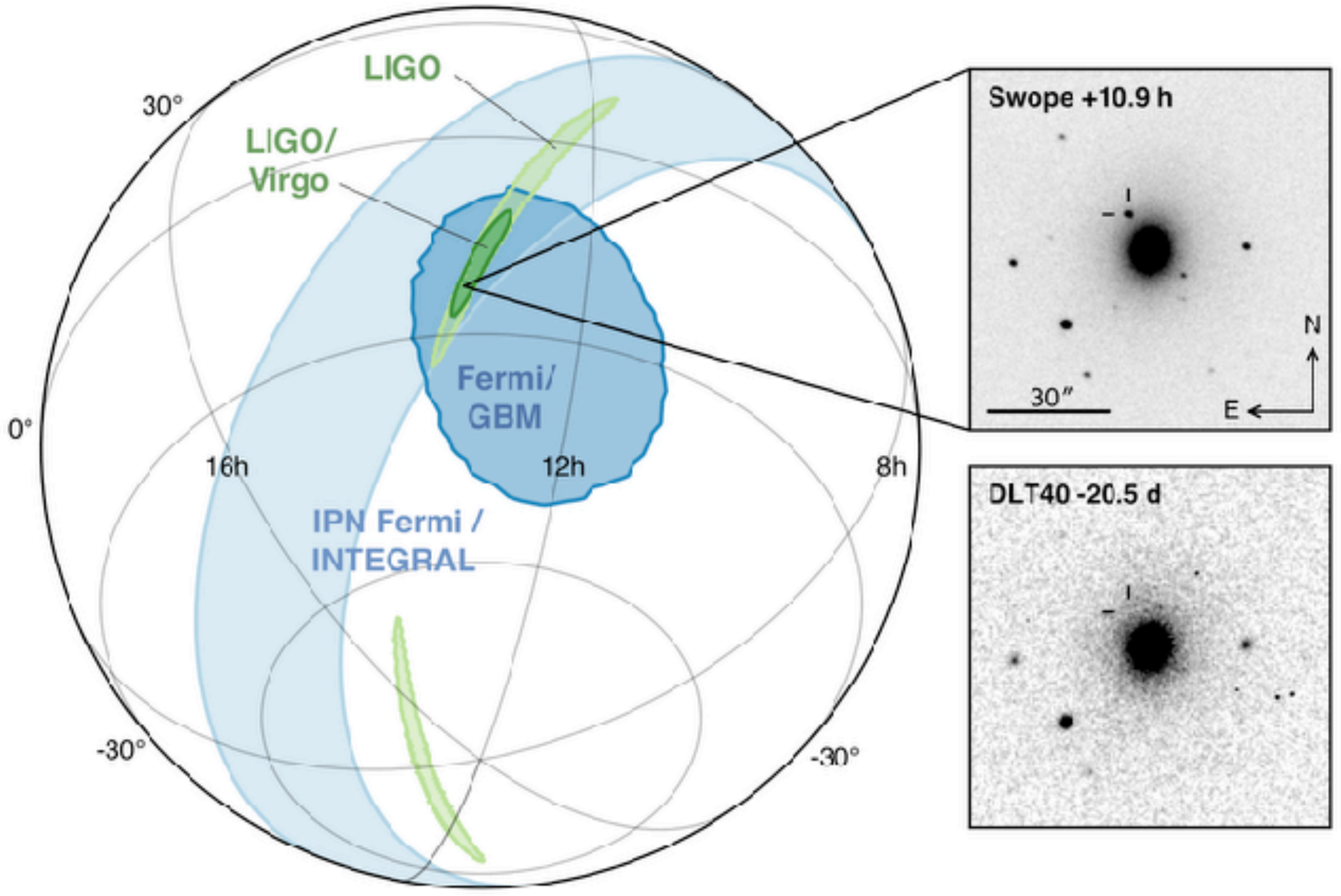


[Sathyaprakash&Schutz, LRR]

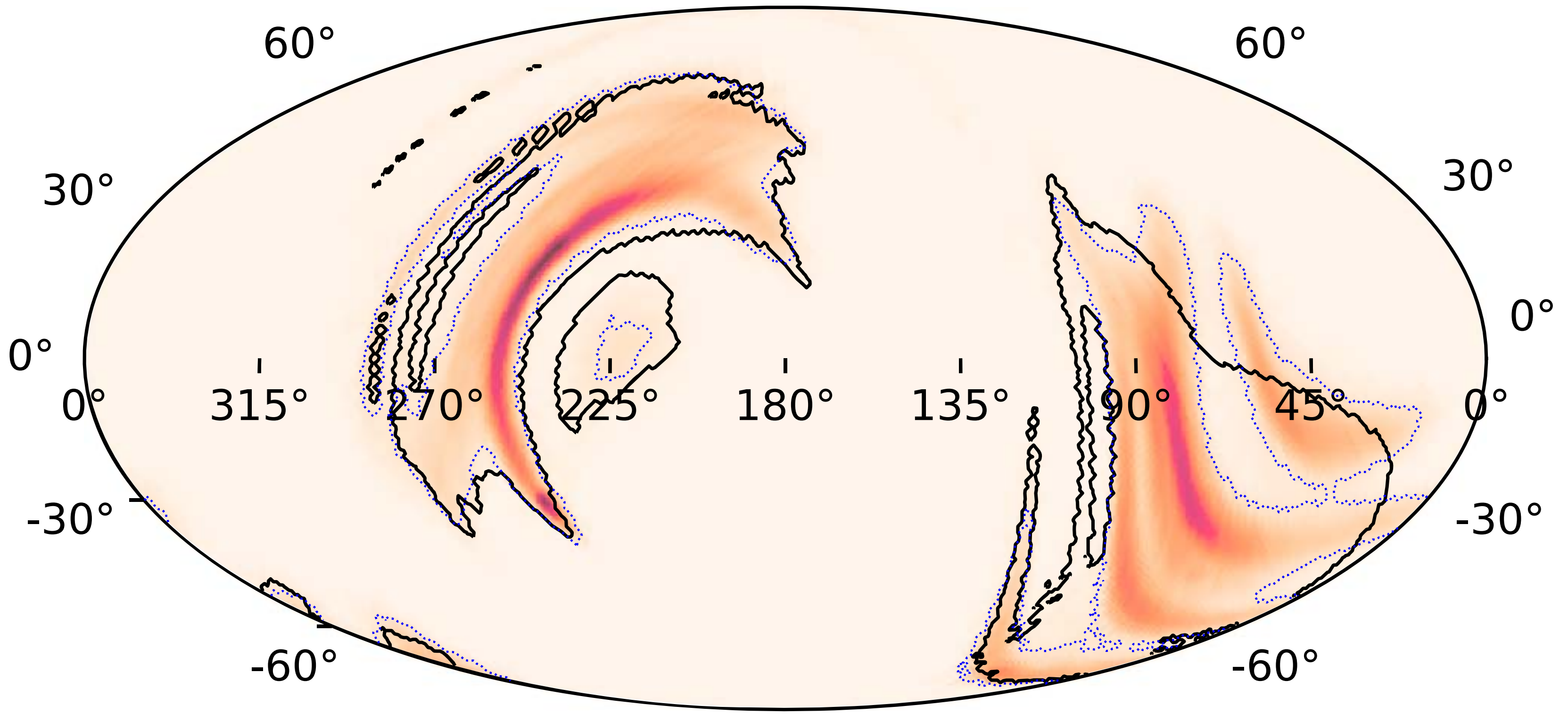




GW170817



GW190425



90% CI ~ 8300 deg^2

[LVK Observing Scenarios & LVK discovery papers]

SOME TAKE-AWAY POINTS

- Gravitational waves are propagating oscillations of a gravitational field generated by accelerating masses
  - They change the proper separation between freely-falling test bodies
  - GWs carry energy & linear momentum from the source
- Spacetime is stiff - "extreme" events are needed to produce a measurable strain
  - Compact binary mergers, CCSNe, rotating neutron stars, stochastic GW background
- Operating GW detectors are km-scale (sophisticated) Michelson interferometers
  - The sensitivity is characterised by the noise power spectral density
  - Current generation can measure length changes $\delta L \simeq 10^{-18} m$
  - Single GW antennae have almost no directional sensitivity; to localise a source a network is needed