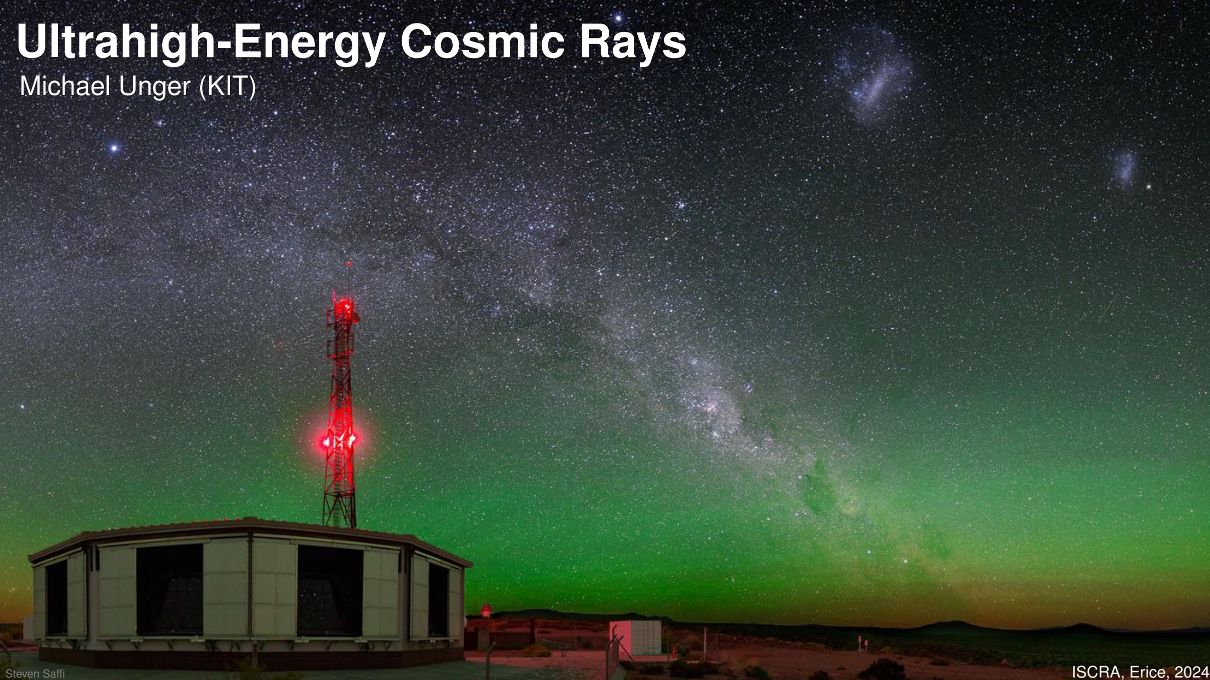
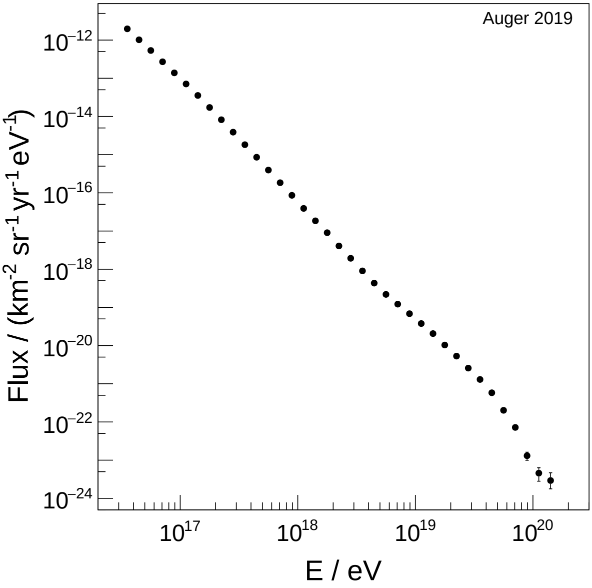
Ultrahigh-Energy Cosmic Rays

Michael Unger (KIT)



Steven Saffi

Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



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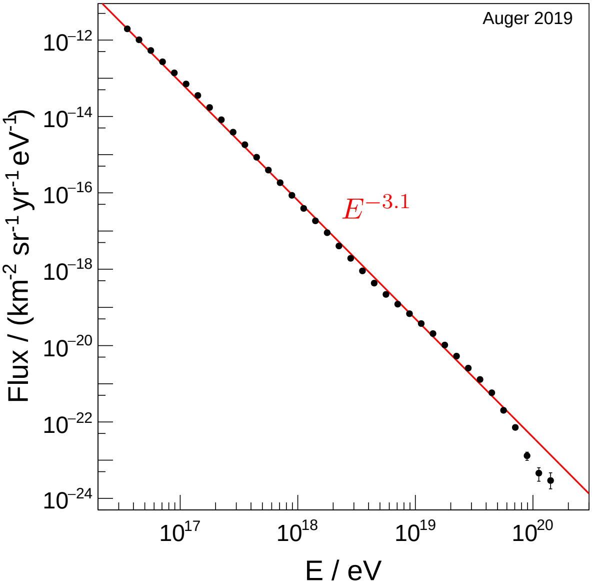

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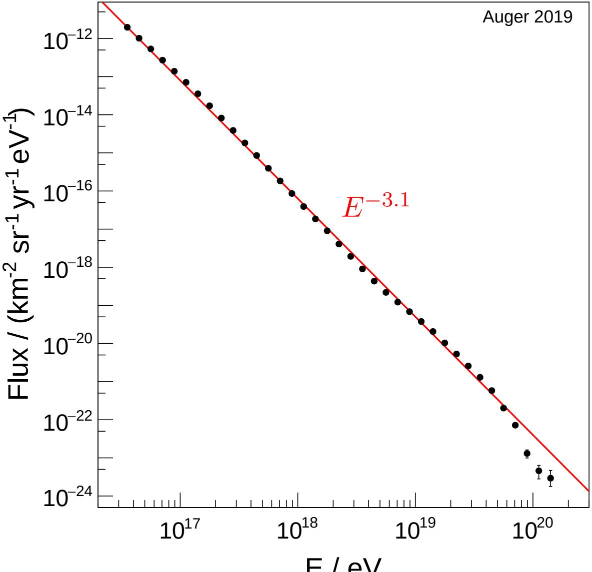

$E_{\text{beam}}^{\text{LHC}} = 7 \times 10^{12}$ eV

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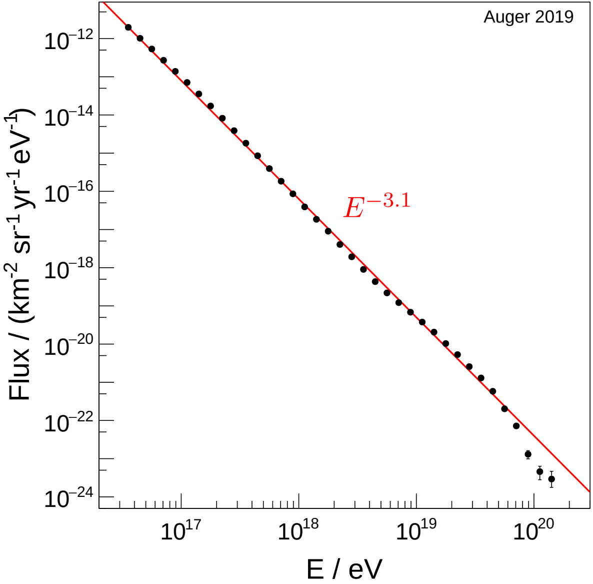


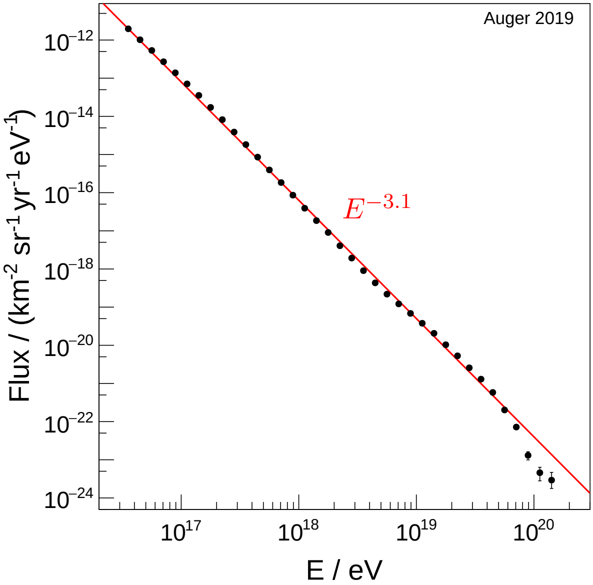
$E_{\rm kin} \sim 4$ TeV

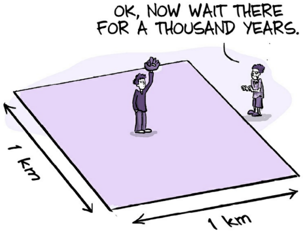
$E^{\rm LHC}_{\rm beam} = 7\times10^{12}$ eV

$\sim$20 J!

Serena Williams' 2nd serve

Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



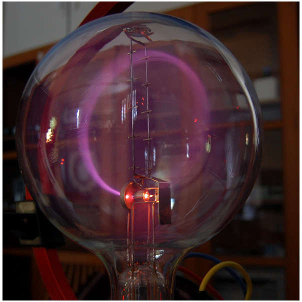


J. Cham&D. Whiteson "We have no idea", Penguin, 2018

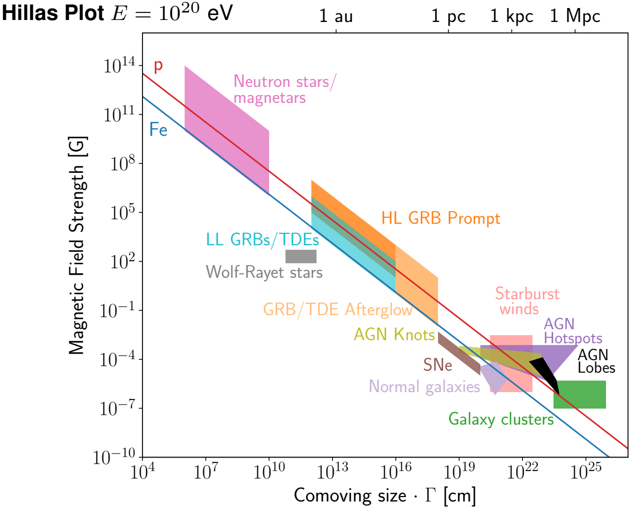
$E_{\text{beam}}^{\text{LHC}} = 7 \times 10^{12}$ eV

ZeVatrons?

magnetic confinement in accelerator



using LHC magnets:



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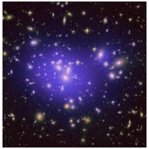

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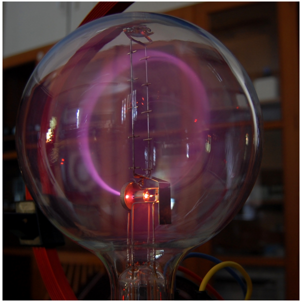
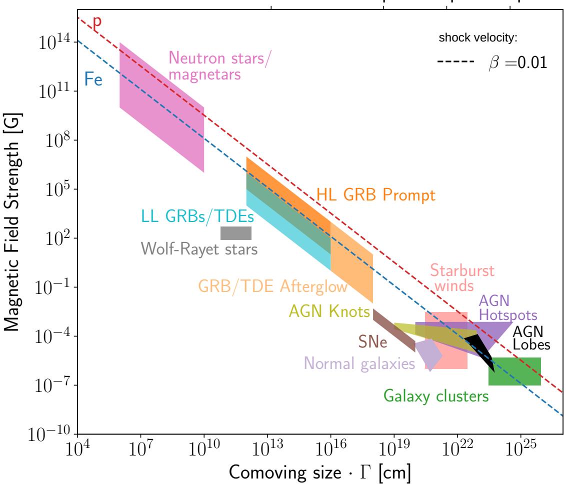
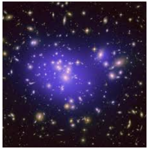


using LHC magnets:

Hillas Plot $E = 10^{20}$ eV







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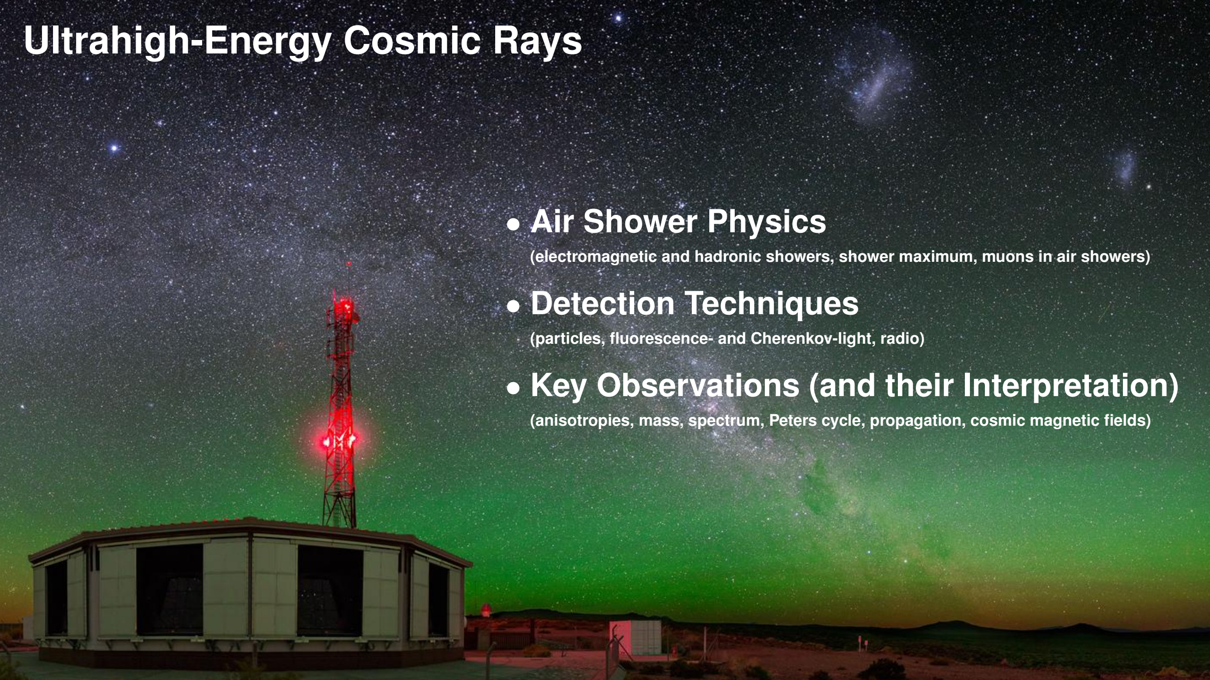
Ultrahigh-Energy Cosmic Rays



- **Air Shower Physics**
  (electromagnetic and hadronic showers, shower maximum, muons in air showers)
- **Detection Techniques**
  (particles, fluorescence- and Cherenkov-light, radio)
- **Key Observations (and their Interpretation)**
  (anisotropies, mass, spectrum, Peters cycle, propagation, cosmic magnetic fields)

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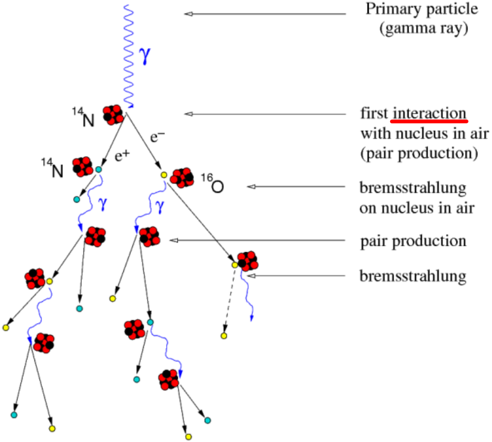
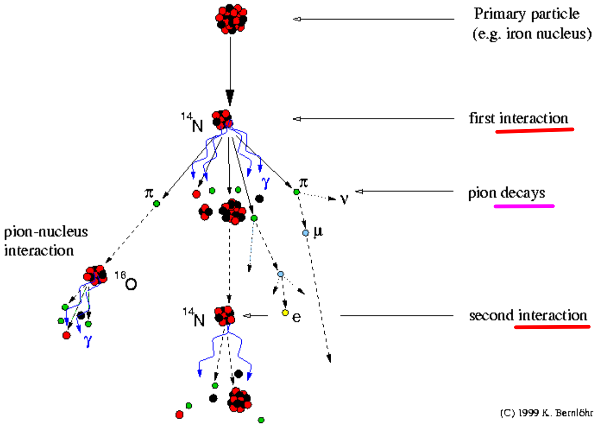
Particle Cascade in the Atmosphere / Air Shower





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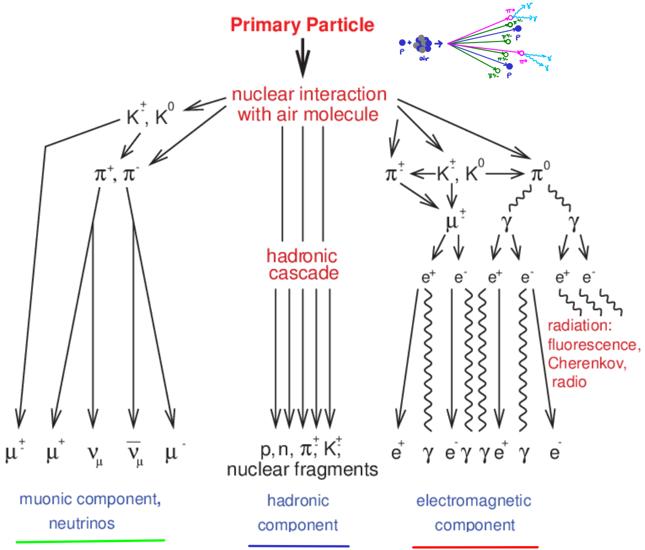
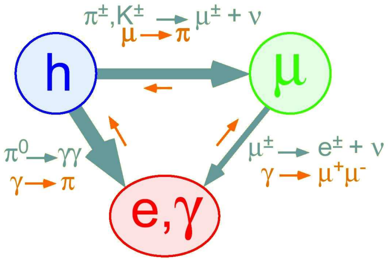
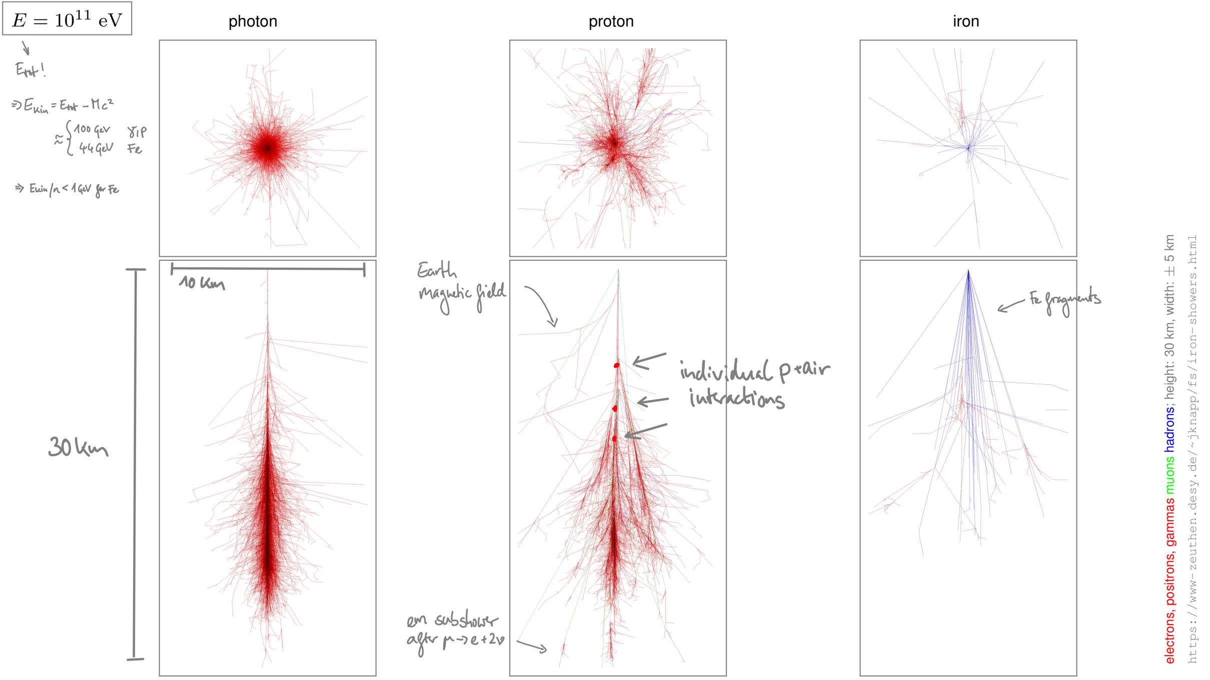




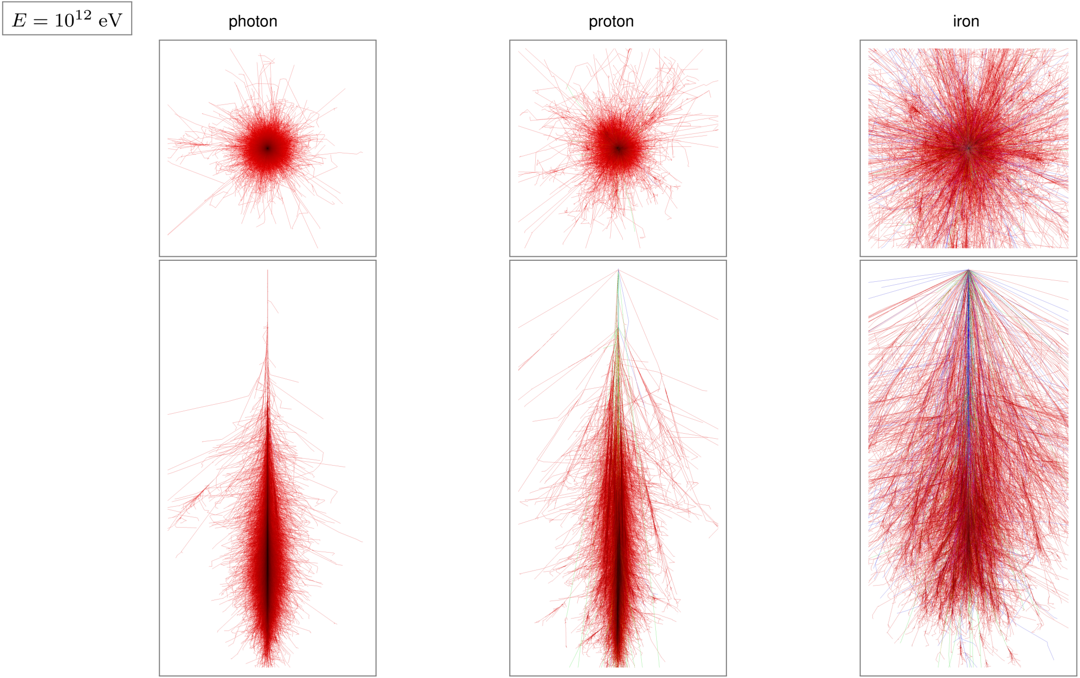
⇒ complicated coupled particle transport through atmosphere

⇒ numerical solutions or Monte Carlo

e.g. CORSIKA (dev. at IAP!)



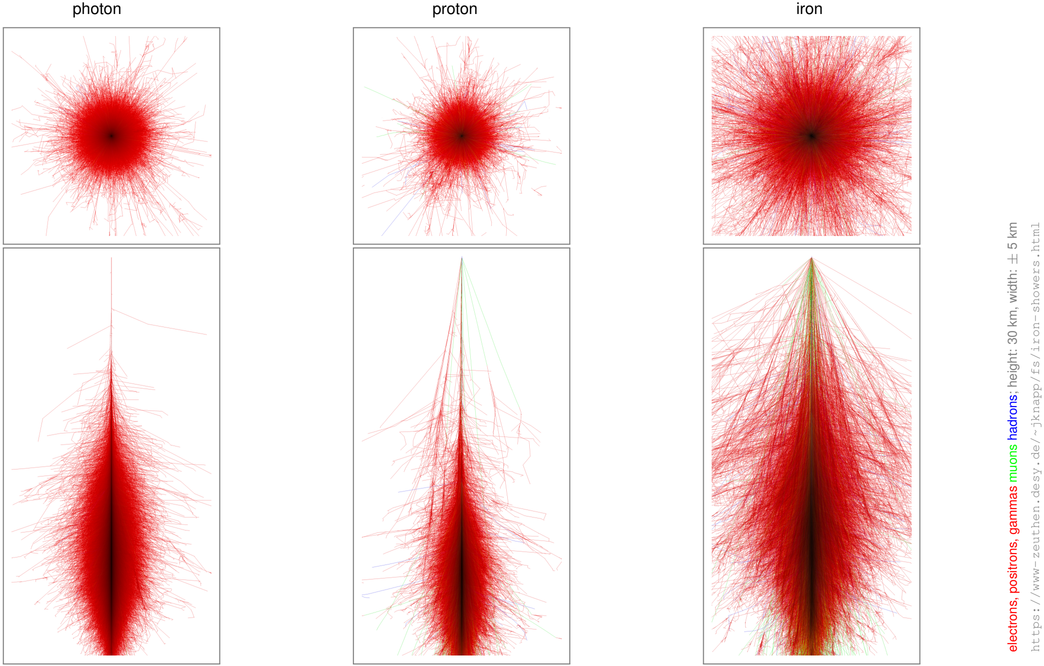
E = 10^11 eV
photon
proton
iron
E_tot!
⇒ E_kin = E_tot − Mc²
≈ 100 GeV γ, p
44 GeV Fe
⇒ E_kin/n < 1 GeV for Fe
10 km
30 km
Earth magnetic field
individual p+air interactions
em subshower after μ → e + 2ν
Fe fragments
electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km
https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html



electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

$E = 10^{13}$ eV


photon
proton
iron
electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km
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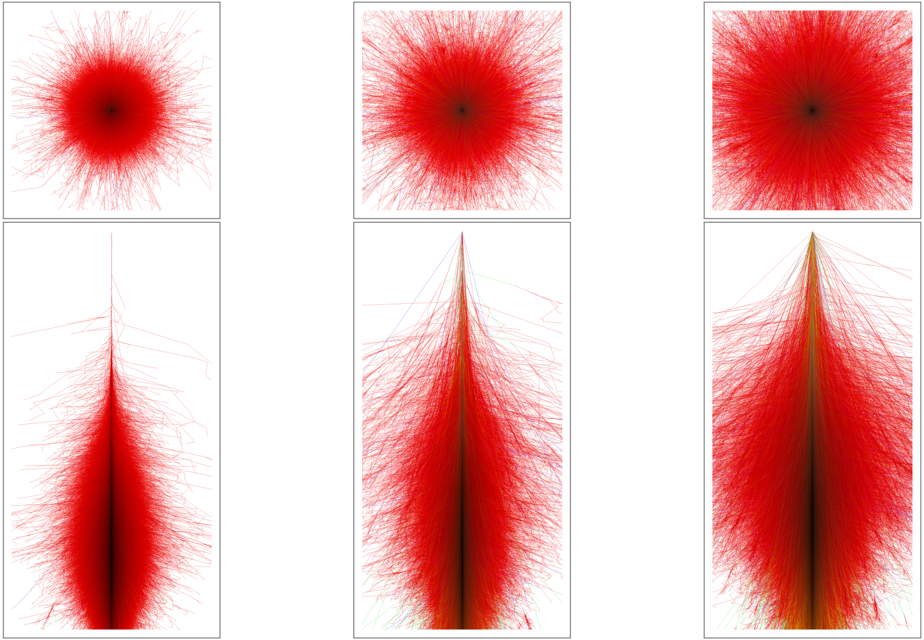
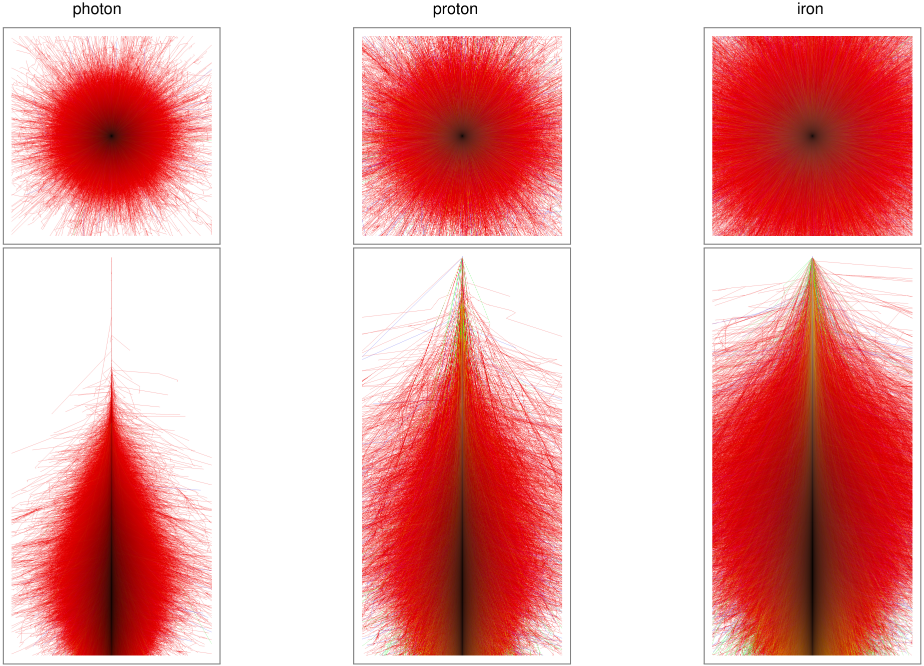
$E = 10^{14}$ eV



electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km
https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

$E = 10^{15}$ eV



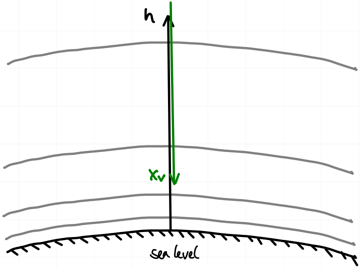
electrons, positrons, gammas muons hadrons; height: 30 km, width: ± 5 km

https://www-zeuthen.desy.de/~jknapp/fs/iron-showers.html

Atmosphere

- height above sea level h
- air density $\rho(h)$
- vertical depth X_v

$$X_v = \int_h^\infty \rho(h')\,dh' \qquad [X_v] = g/cm^2 \Rightarrow \text{"grammage"}$$



- isothermal atmosphere:

$$\rho(h) = \rho_0\, e^{-h/h_0}$$
$$X_v = X_0\, e^{-h/h_0}$$

- $X_0 \approx 1030\ g/cm^2$ at sea level
- scale height $h_0 \approx 8.4$ km at sea level, ≈ 6.4 km high altitudes above $h \approx 10$ km

altitude (km) (h)	vertical depth (g/cm^2) (X_v)	local density (10^{-3} g/cm^3) ($\rho(h)$)	Molière unit (m) (lateral spread due to Coulomb scattering)	Cherenkov threshold (MeV) (see lecture 2)	Cherenkov angle (°) (see lecture 2)
40	3	3.8×10^{-3}	2.4×10^4	386	0.076
30	11.8	1.8×10^{-2}	5.1×10^3	176	0.17
20	55.8	8.8×10^{-2}	1.0×10^3	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1032	1.23	76	21	1.36

Atmosphere

- slant depth:

$$X = \int_{\ell}^{\infty} \rho(h(\ell'))\, d\ell'$$

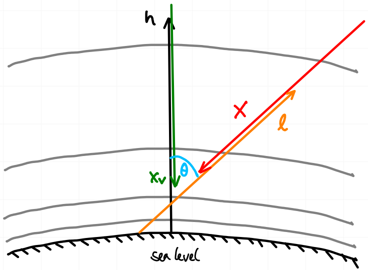
- zenith angle θ $\quad h/\ell = \cos\theta$

- flat atmosphere approximation for $\theta \lesssim 65°$

$$X = X_v / \cos\theta$$

- horizontal thickness of curved atmosphere:

$$X(\theta = 90°) \approx 3.5 \cdot 10^4\ g/cm^2$$



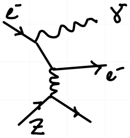
zenith angle degree	planar distance km	planar slant depth g/cm²	spherical distance km	spherical slant depth g/cm²
0	112.8	1036.1	112.8	1036.1
30	130.3	1196.4	129.9	1196.0
45	159.6	1465.3	158.2	1463.7
60	225.7	2072.2	220.1	2065.3
70	329.9	3029.4	310.7	3003.9
80	649.8	5966.7	529.0	5765.9
85	1294.6	11887.9	770.9	10572.1
89	6465.0	59367.2	1098.3	25920.4
90	∞	∞	1204.4	36481.8

Table 1: Distances and slant depths in planar and spherical geometry, calculated with the Linsley parametrization of the U.S. standard atmosphere.

D. Heck FZKA-6954 2004

Electromagnetic Interactions

interactions with nuclei of material (Z)



bremsstrahlung

energy loss

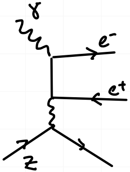
$$\left\langle -\frac{dE}{dx} \right\rangle_{brems,pair} = \frac{E}{X_0} \quad \leftrightarrow \quad E(x) = E_0 e^{-x/X_0}$$

radiation length:

$$X_0 \sim \left(\underbrace{\frac{Z^2}{A} \rho}_{\text{material}} \right)^{-1}$$

⇒ electron radiation length in air:

$$\boxed{X_0^{air} = 36.6\ g/cm^2}$$



pair production

critical energy:

$$\left\langle -\frac{dE}{dx} \right\rangle_{brems} \sim E$$

$$\left\langle -\frac{dE}{dx} \right\rangle_{ion} \sim \text{const}$$

E_{crit} when $\left\langle -\frac{dE}{dx} \right\rangle_{brems} = \left\langle -\frac{dE}{dx} \right\rangle_{ion}$

⇒ critical energy in air:

$$\boxed{E_{crit}^{air} = 84\ MeV}$$

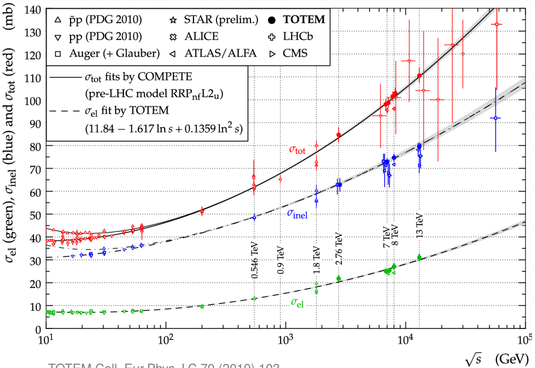
Hadronic Interactions

- charge radius (e^+p scattering):

$$r_p = 0.88 \cdot 10^{-15}\ m$$

$\rightarrow \sigma_{pp} \approx (2r_p)^2 \pi \approx 100\,mb$

(b: "barn", $1b = 10^{-28} m^2$)



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- inelastic cross section: $\sigma_{inel} = \sigma_{tot} - \sigma_{ela}$

  ≙ particle production — total — elastic

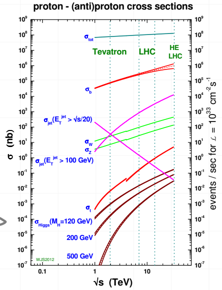
$$\sigma_{inel} \approx 35\,mb$$

$(10\,GeV < E_{lab} < 1\,TeV)$

- particle production: $p + p \rightarrow p + p + m \cdot \pi^{+/-} + n \cdot \pi^0$

(and $K^{+/-}$, Λ, K^0, n, Higgs ...)

but...



- pion multiplicity: $m \approx 2 \cdot n$

Hadronic Interactions

- interaction length: $j + \text{air} \to X$

$$\lambda_j = \ell_j \, \rho = \frac{\rho}{n_A \, \sigma_j^{air}} = \frac{\langle A \rangle m_p}{\sigma_j^{air}}$$

mass density (ρ); $[\lambda] = g/cm^2$; $[\ell] = cm$; number density (n_A); cross section (σ_j^{air})

- typical values: @ 10 TeV

$\lambda_N \approx 80\ g/cm^2$ p+air / n+air

$\lambda_\pi \approx 100\ g/cm^2$ π+air

- average air mass: $\langle A \rangle = 14.6$ (78.09% N, 20.95% O, 0.93% Ar)
- nucleon + nucleus interactions:

$$\sigma(p+A) \sim A^{2/3}$$

← geometrical size of nucleus with A spherically packed nucleons

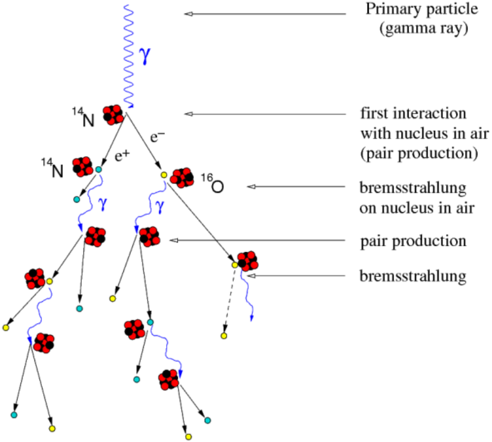
- nucleus + nucleus interactions:

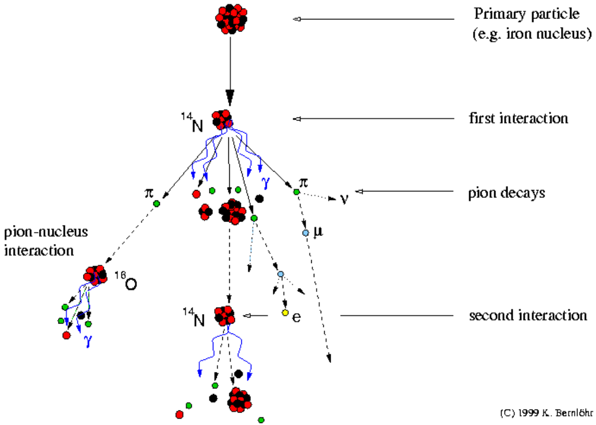
$$\sigma(A_1 + A_2) \approx \pi R_0^2 \left(A_1^{1/3} + A_2^{1/3} - \delta \right)^2$$

($\delta = 1.12$, $R_0 = 1.47$ fm)

- Glauber model of h+A scattering (see CRPP A6 and Glauber + Matthiae Nucl. Phys. B 21 (1970) 135)

Particle Cascade in the Atmosphere / Air Shower

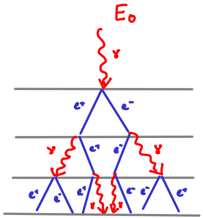




Photon-induced Shower

Heitler model

γ, e^-, e^+ shower



	N_n	E_n
$n = 0$	1	E_0
$n = 1$	2	$E_0/2$
$n = 2$	4	$E_0/4$
$n = 3$	8	$E_0/8$
⋮	⋮	⋮

Carlson + Oppenheimer 1937, Heitler 1954

Photon-induced Shower

- radiation length X_0 in air: $37\,g/cm^2$

$$\left\langle -\frac{dE}{dx}\right\rangle_{brems,pair} = \frac{E}{X_0} \quad \leftrightarrow \quad E(x) = E_0 e^{-x/X_0}$$

- splitting length $d = \ln 2 \cdot X_0$ $\quad E(d) = E_0/2$
  - $E_{i+1} \to E_i/2$
  - $N_{i+1} \to 2 \cdot N_i$

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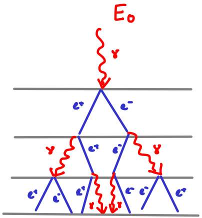

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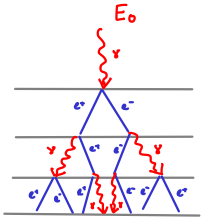
  - $E_{i+1} \rightarrow E_i/2$
  - $N_{i+1} \rightarrow 2 \cdot N_i$
- after n splitting lengths:
  - $X_n = n \ln 2 \, X_0$
  - $N_n = 2^n = e^{X/X_0}$
  - $E_n = E_0/N_n$

Heitler model

γ, e^-, e^+ shower



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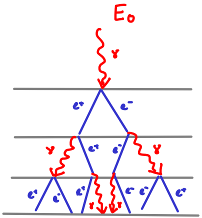

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⋮	⋮	⋮

Carlson + Oppenheimer 1937, Heitler 1954

critical energy:

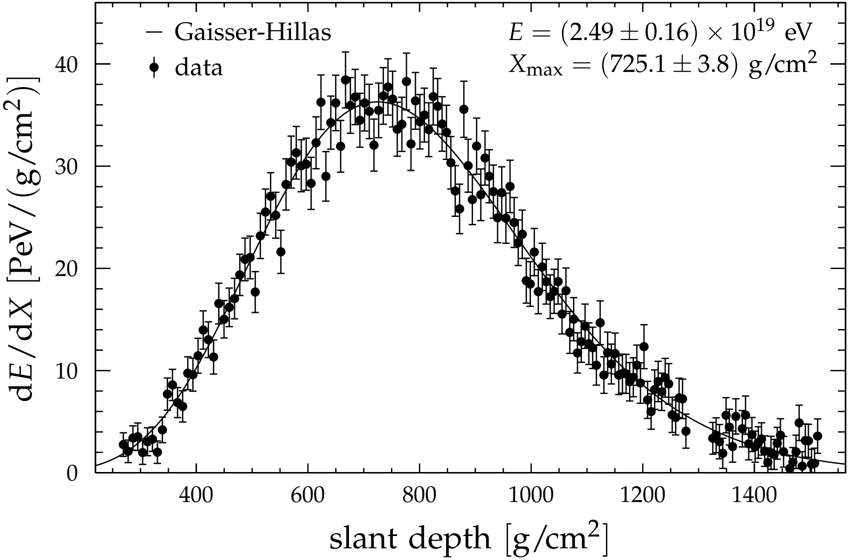
$$\frac{dE_{rad}}{dX}(\varepsilon_{crit}) = \frac{dE_{ion}}{dX}(\varepsilon_{crit})$$

in air: 84 MeV

- shower development stops when $E_n \leq \varepsilon_{crit}$
  - $N_{max} = E_0/\varepsilon_{crit} \quad = 10^{11}$
  - $n_{max} = \ln(E_0/\varepsilon_{crit})/\ln 2 \quad = 37$
  - $X_{max} = X_0 \ln(E_0/\varepsilon_{crit}) \quad = 940\ g/cm^2$

$\Big\}\ E_0 = 10^{19}$

Example of a Measured Longitudinal Profile not a photon!



MIP: $\mathrm{d}E/\mathrm{d}X \sim 2$ MeV/(g/cm^2) $\rightarrow N_{max} \sim 40 \times 10^{15}/2 \times 10^6 = 2 \times 10^{10}$

Pierre Auger Coll., PRD 2014

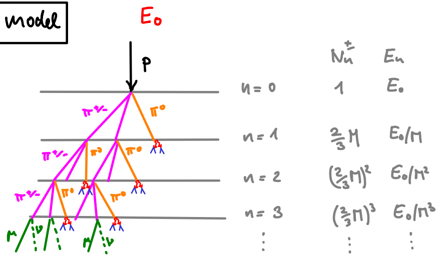
Proton-induced Shower

$M = M_- + M_+ + M_0\ ,\ M_- \approx M_+$

- production of M pions, $p + air \rightarrow M_- \cdot \pi^- + M_+ \cdot \pi^+ + M_0 \cdot \pi^0 + \ldots$
  (M: multiplicity)

Matthews-Heitler model

hadronic shower



J. Matthews APP 2005

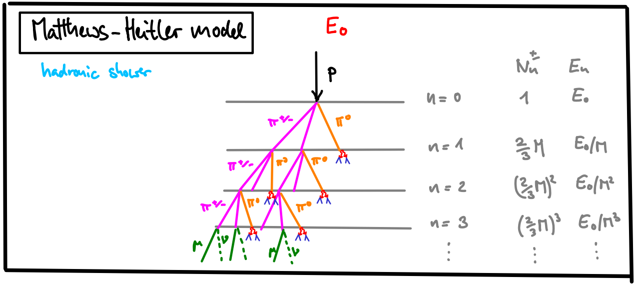
Proton-induced Shower

$M = n_- + n_+ + n_0$, $n_- \approx n_+$

- production of M pions, $p + \text{air} \rightarrow n_- \cdot \pi^- + n_+ \cdot \pi^+ + n_0 \cdot \pi^0 + \ldots$
  (M: multiplicity)
- interaction length λ_{int}
  - $E_{i+1} \rightarrow E_i / M$, $E_n = E_0 / M^n$
- $\pi^{+/-} + \text{air} \rightarrow \frac{2}{3} M \cdot \pi^{\pm} + \frac{1}{3} M \pi^0$

re-interaction → hadronic shower

decay: $\pi^0 \rightarrow \gamma\gamma$ → electromagnetic shower



J. Matthews APP 2005

Proton-induced Shower

$M = M_- + M_+ + M_0\,,\ M_- \approx M_+$

- production of M pions, $p + air \to M_- \cdot \pi^- + M_+ \cdot \pi^+ + M_0 \cdot \pi^0 + \ldots$
  (M: multiplicity)
- interaction length λ_{int}
  - $E_{i+1} \to E_i / M$, $E_n = E_0 / M^n$
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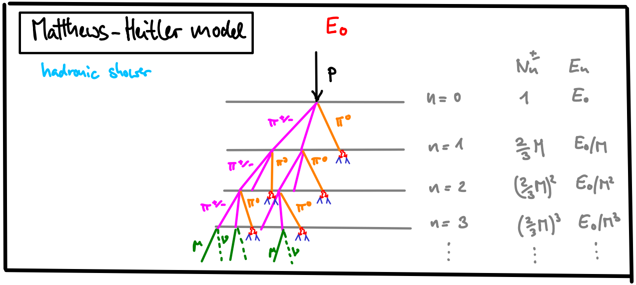
- hadronic cascade stops when $\lambda_{int} = \lambda_{dec}$ $\quad \pi^+ \to \mu^+ \nu_\mu$
  $(\lambda_{dec} = \beta \gamma c \tau)$ $\quad \pi^- \to \mu^- \bar{\nu}_\mu$
  → critical energy $\epsilon_\pi \approx 10\,GeV = E_n \to n_{crit} = \ln(E_0/\epsilon_\pi) / \ln M$
- $N_n = \left(\frac{2}{3} M\right)^n$



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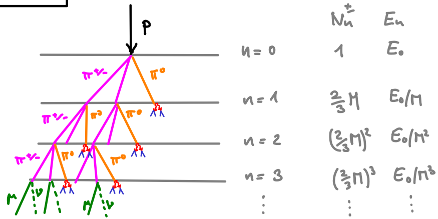
Matthews-Heitler model



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$$N_\mu = N_{n_{crit}} = \left(\frac{E_0}{\varepsilon_\pi}\right)^\beta$$

with $\beta = \frac{\ln \frac{2}{3} M}{\ln M} = 1 - \frac{\ln 3/2}{\ln M} = 1 - \frac{\lg 3/2}{\lg M} \approx 1 - \frac{0.18}{\lg M}$

≈ 0.9 for $M = 50$

e.g. $N_\mu \approx 10^8$ for $E_0 = 10^{19}\,\mathrm{eV}$ and $M = 50$

Proton-induced Shower

estimate of shower maximum:

- photons produced in π^0 decays after first interaction:

  $\frac{1}{3}M$ π^0's with $E = E_0/M \Rightarrow 2 \cdot \frac{1}{3}M$ photons with $E_\gamma = E_0/M/2$

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- photons produced in π^0 decays after first interaction:

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- $2 \cdot \frac{1}{3}M$ electromagnetic showers starting at $\langle x_1 \rangle = \lambda_p$

  $$\langle X^p_{max} \rangle = \lambda_p + X^\gamma_{max}\left(E = \frac{E_0}{2M}\right)$$

$$\Rightarrow \boxed{\langle X^p_{max} \rangle = \lambda_p + X_0 \ln\left(\frac{E_0}{2 \cdot M \cdot \varepsilon_c}\right)}$$

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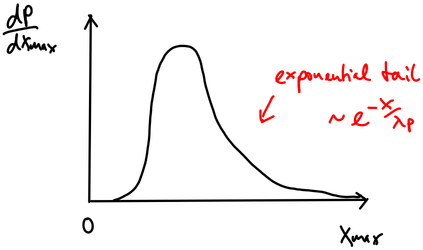
$\Rightarrow$ X_{max} distribution

$$\underbrace{}_{}\frac{dp}{dX_{max}} = \underbrace{\frac{dp}{dX_1}}_{\text{1st interaction}} \otimes \underbrace{\frac{dp}{d\Delta X}}_{\text{shower development}} \qquad \Delta X = X_{max} - X_1$$

(exp) ⊗ (≈ gauss)

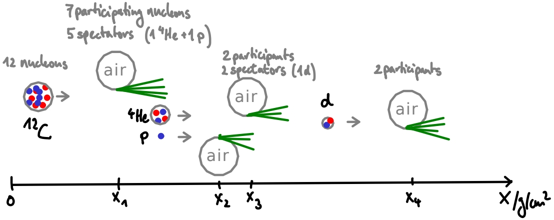


$\Rightarrow$ measurement of $\sigma_{p\text{-air}}$ @ $\sqrt{s} \approx 100$ TeV $\gg \sqrt{s}_{LHC}$

Nucleus-induced Shower

Superposition model $(E, A) + \text{air} \rightarrow X \;\hat{=}\; A \cdot (E/A, 1) + \text{air} \rightarrow X$

but e.g.:



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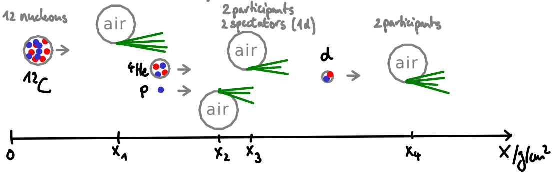
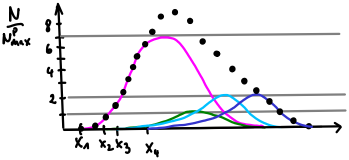


$\rightarrow p(X_{i+1}-X_i) \sim e^{-\frac{X_{i+1}-X_i}{\lambda_{AN}}}$

→ superposition of 12 N+air showers from 4 A_N+air interactions: 7N@X_1, 1N@X_2, 2N@X_3, 2N@X_4



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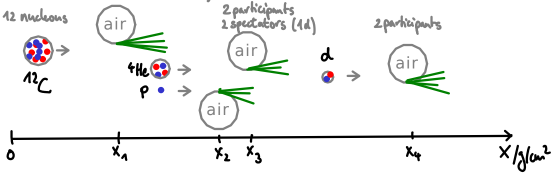

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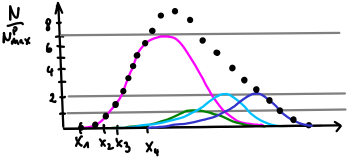


Superposition theorem J. Engel et al. PRD 1992

if average number of participating nucleons in projectile A+air interactions

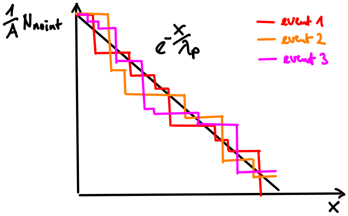
$$\langle n_A \rangle = A\,\frac{\lambda_A}{\lambda_p} \qquad \langle n_A\rangle = \sum n\,P(n)$$

then probability of depth of interaction of nucleons

$$\frac{dp_A}{dX} = \frac{1}{\lambda_p}\, e^{-X/\lambda_p}$$

irrespective of fragmentation of spectator nucleus

number of nucleons that have not yet interacted $\frac{1}{A} N_{noint} = 1 - \int_0^X \frac{dp_A}{dX'}\,dX'$
$= \exp(-X/\lambda_p)$



Nucleus-induced Shower

- number of muons: $N_\mu = A \cdot \left(\frac{E_0/A}{\varepsilon_\pi}\right)^\beta = \left(\frac{E_0}{\varepsilon_\pi}\right)^\beta A^{1-\beta}$

e.g. $M=50$, $\beta=0.9$

$\rightarrow$ $N_\mu(56) / N_\mu(1) = 56^{0.1} = 1.5$

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$$X^A_{max}(E) = X^p_{max}(E/A)$$

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$$X_{max}^A(E) = X_{max}^p(E/A)$$

how to measure the CR mass + energy with air showers

measurement		primary CR	detector
N_e, N_μ	↔	E_0, A	SD
X_{max}, E_{em}	↔	E_0, A	FD/CD
E_{em}, N_μ	↔	E_0, A	RD+SD
$N_e, N_\mu, X_{max}, E_{em}$	↔	E_0, A	SD+FD

"hybrid" (RD+SD, SD+FD)

overconstrained ⇒ check hadronic interaction models

(SD: surface detector (particles)
FD: fluorescence detector
CD: Cherenkov detector
RD: radio detector)

Ultrahigh-Energy Cosmic Rays

- **Air Shower Physics**
  (electromagnetic and hadronic showers, shower maximum, muons in air showers)
- **Detection Techniques**
  (particles, fluorescence- and Cherenkov-light, radio)
- **Key Observations (and their Interpretation)**
  (anisotropies, mass, spectrum, Peters cycle, propagation, cosmic magnetic fields)