UNIVERSITY OF BIRMINGHAM | GRAVITATIONAL WAVE ASTRONOMY

LECTURE 2: GRAVITATIONAL-WAVE DATA ANALYSIS FOR COMPACT BINARIES

PATRICIA SCHMIDT

ISCRA 2024

SUMMARY LECTURE 1

- Gravitational waves are propagating oscillations of a gravitational field **generated by accelerating masses**
  - They **change the proper separation** between freely-falling test bodies
  - GWs **carry energy & momentum** from the source
- Spacetime is stiff - "extreme" events are needed to produce a measurable strain
  - **Compact binary mergers, CCSNe, rotating neutron stars, stochastic GW background**
- Operating GW detectors are km-scale (sophisticated) **Michelson interferometers**
  - The **sensitivity** is characterised by the **noise power spectral density (PSD)**
  - Current generation of GW detectors can measure length changes $\delta L \simeq 10^{-18} m$
  - Observe GWs with frequencies between 20-2000 Hz

DATA ANALYSIS FOR COMPACT BINARIES

GW detection

Parameter estimation

Modelling gravitational waves

(Some) Recommended literature:

★ Luc Blanchet, Living Reviews in Relativity: https://link.springer.com/article/10.12942/lrr-2014-2

★ Buonanno & Sathyaprakash: https://arxiv.org/abs/1410.7832

★ Talbot & Thrane: https://arxiv.org/abs/1809.02293

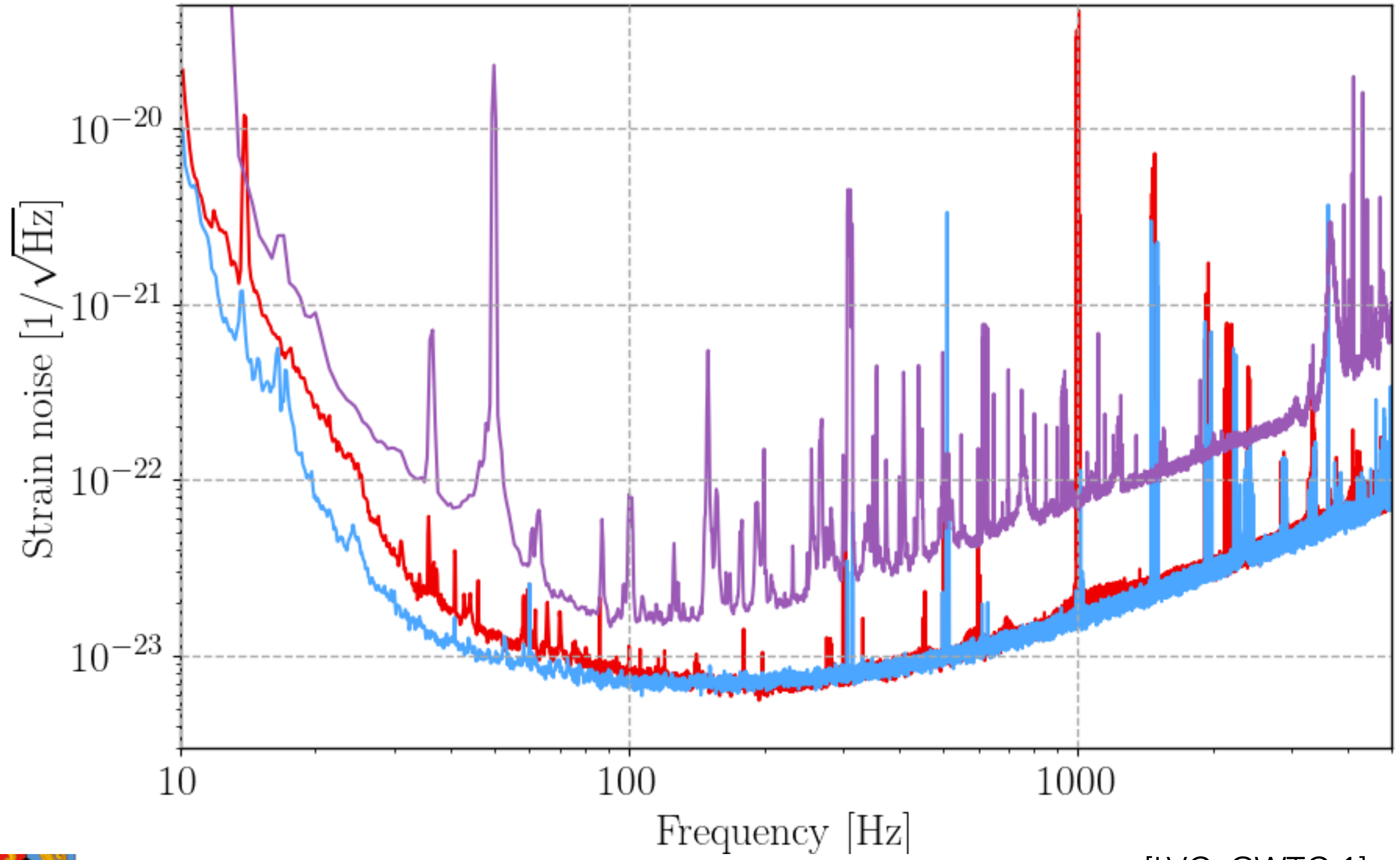
SENSITIVITY

- The sensitivity of GW detector is characterised by the **power spectral density (PSD)** of its noise background in the absence of a GW signal.

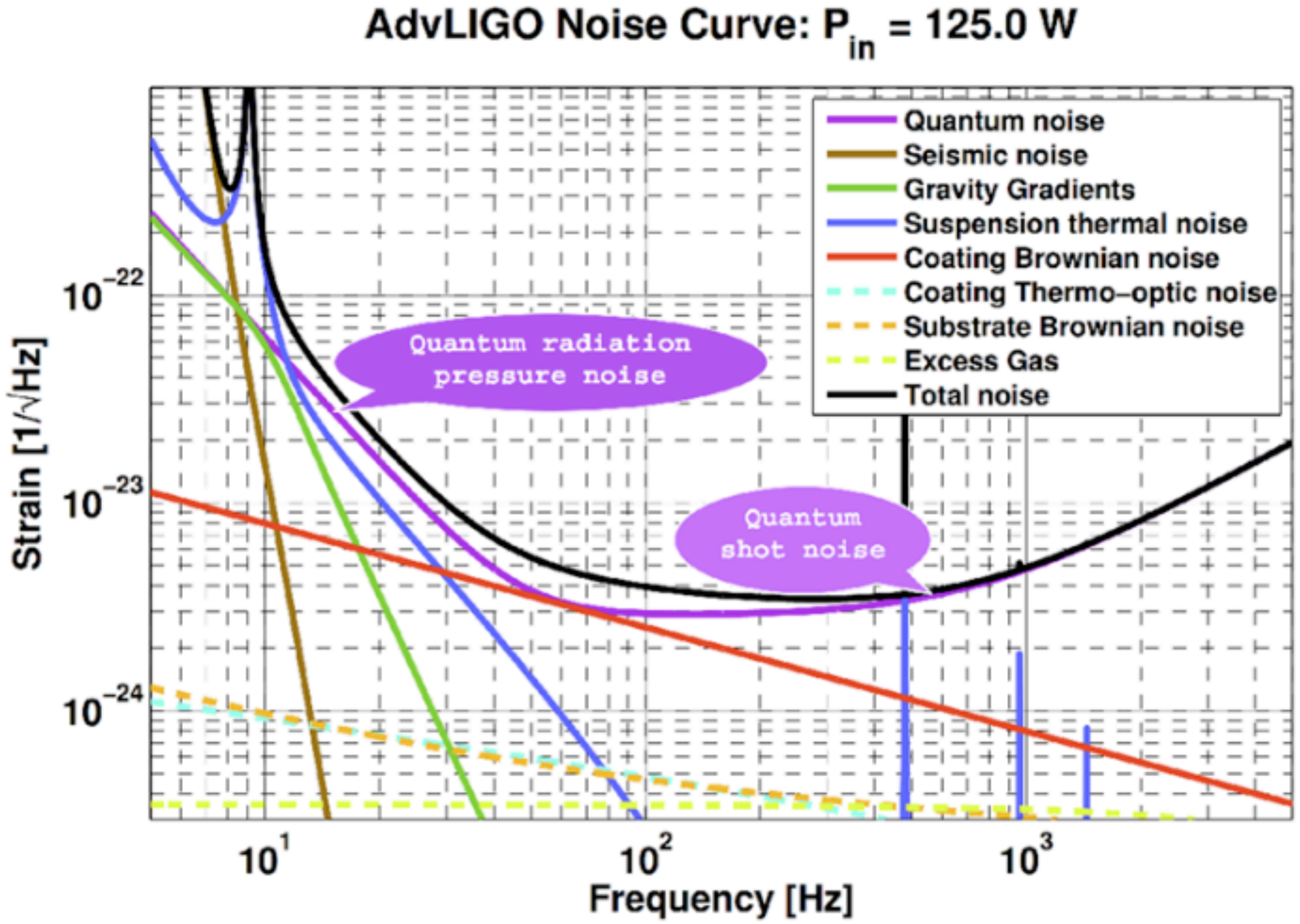
Assuming stationary & Gaussian

- $S_n(f)$ is the **noise power spectral density** - the Fourier transform of the noise autocorrelation function: $\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \frac{1}{2}S_n(f)\delta(f-f')$

Amplitude spectral density = $\sqrt{\text{PSD}}$

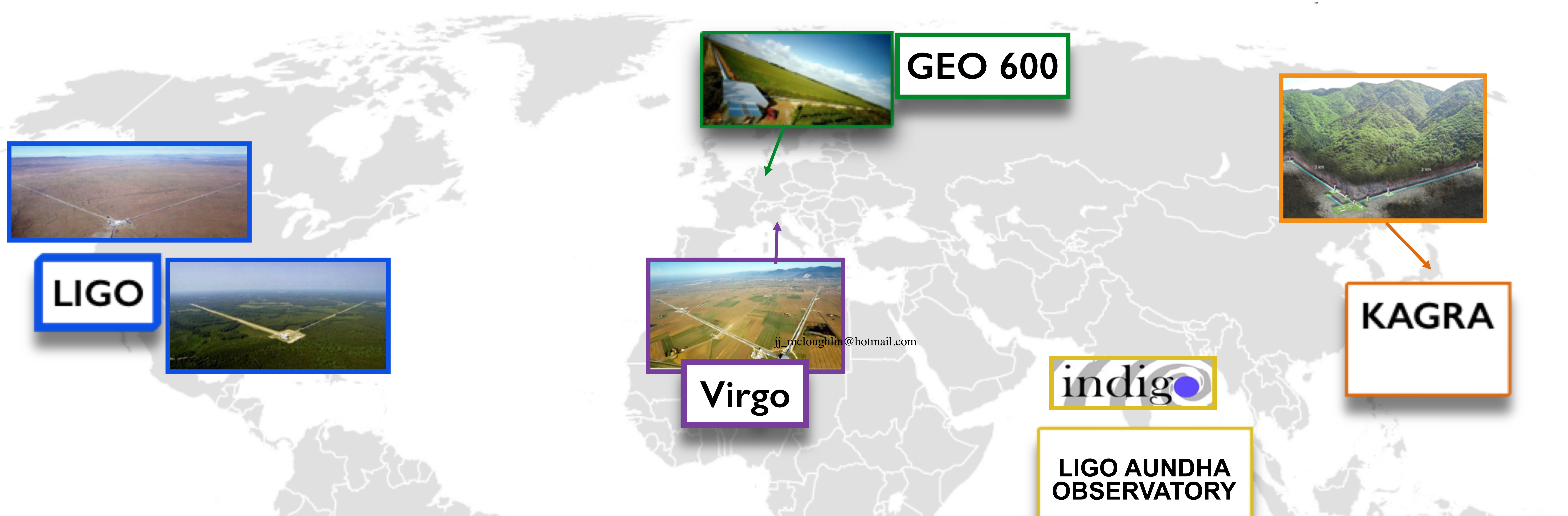


[LVC, GWTC-1]

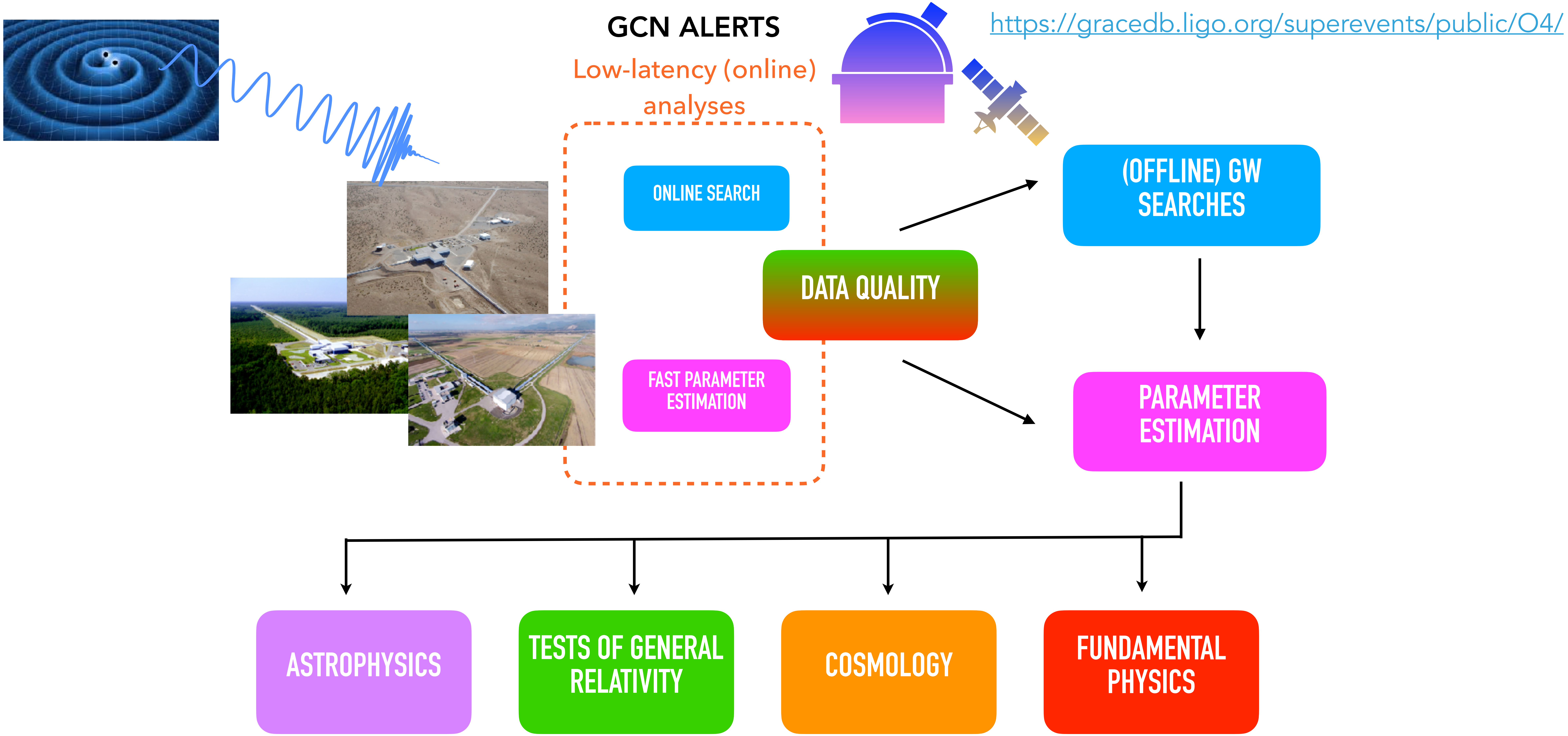


[Living Rev. Rel. 2019 22:2]

DETECTOR NETWORK



- Kilometre-scale interferometres
- Sensitive to GWs between a few Hz to a few kHz
- Simultaneous detection increases detection confidence
- Improved sky localisation & polarisation measurement
- Increased duty cycle

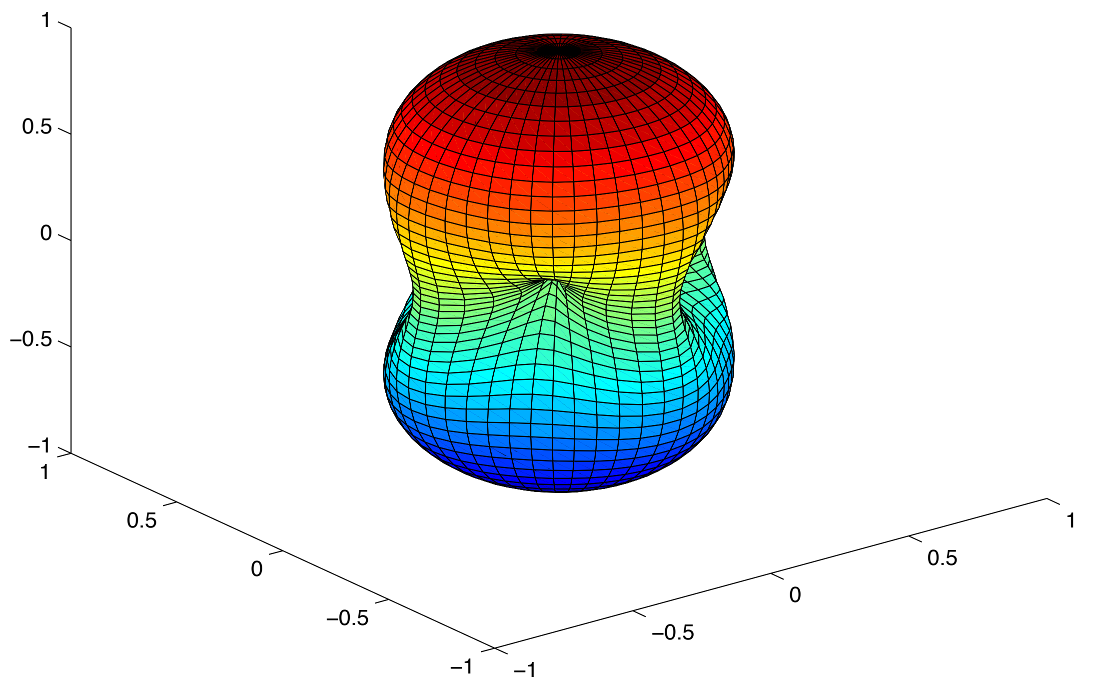

https://gracedb.ligo.org/superevents/public/O4/
GCN ALERTS
Low-latency (online) analyses
ONLINE SEARCH
FAST PARAMETER ESTIMATION
DATA QUALITY
(OFFLINE) GW SEARCHES
PARAMETER ESTIMATION
ASTROPHYSICS
TESTS OF GENERAL RELATIVITY
COSMOLOGY
FUNDAMENTAL PHYSICS

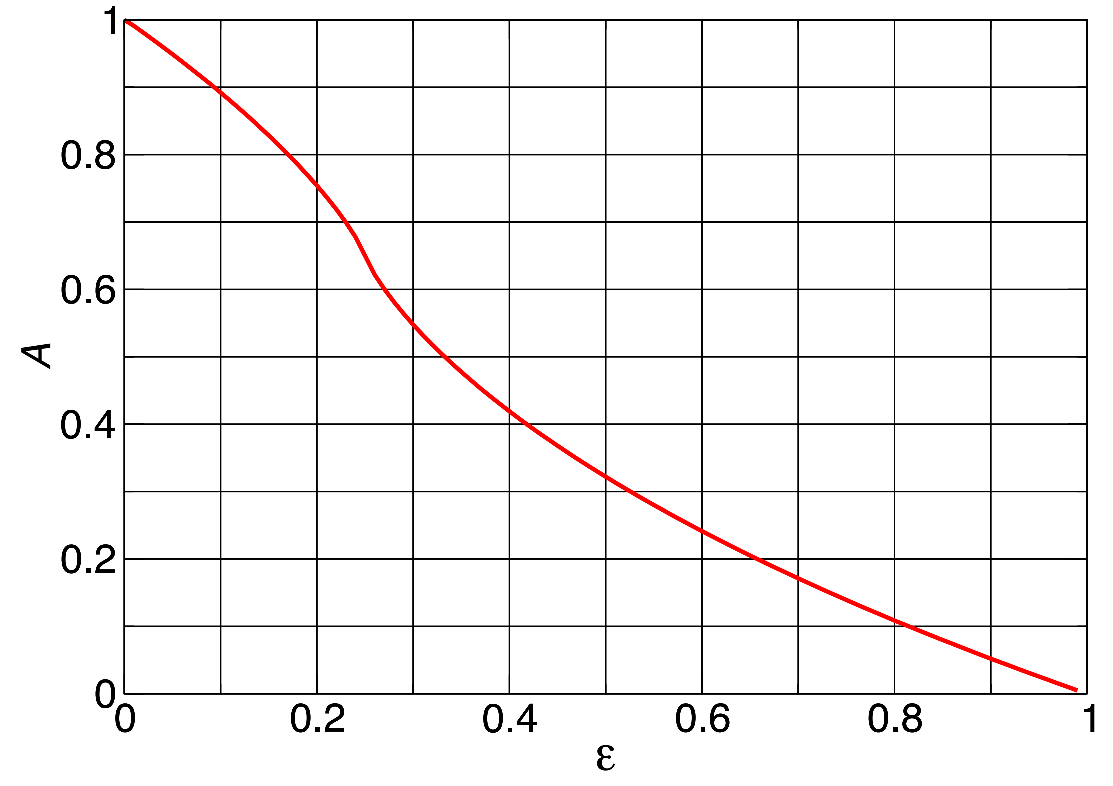
LOCALISATION

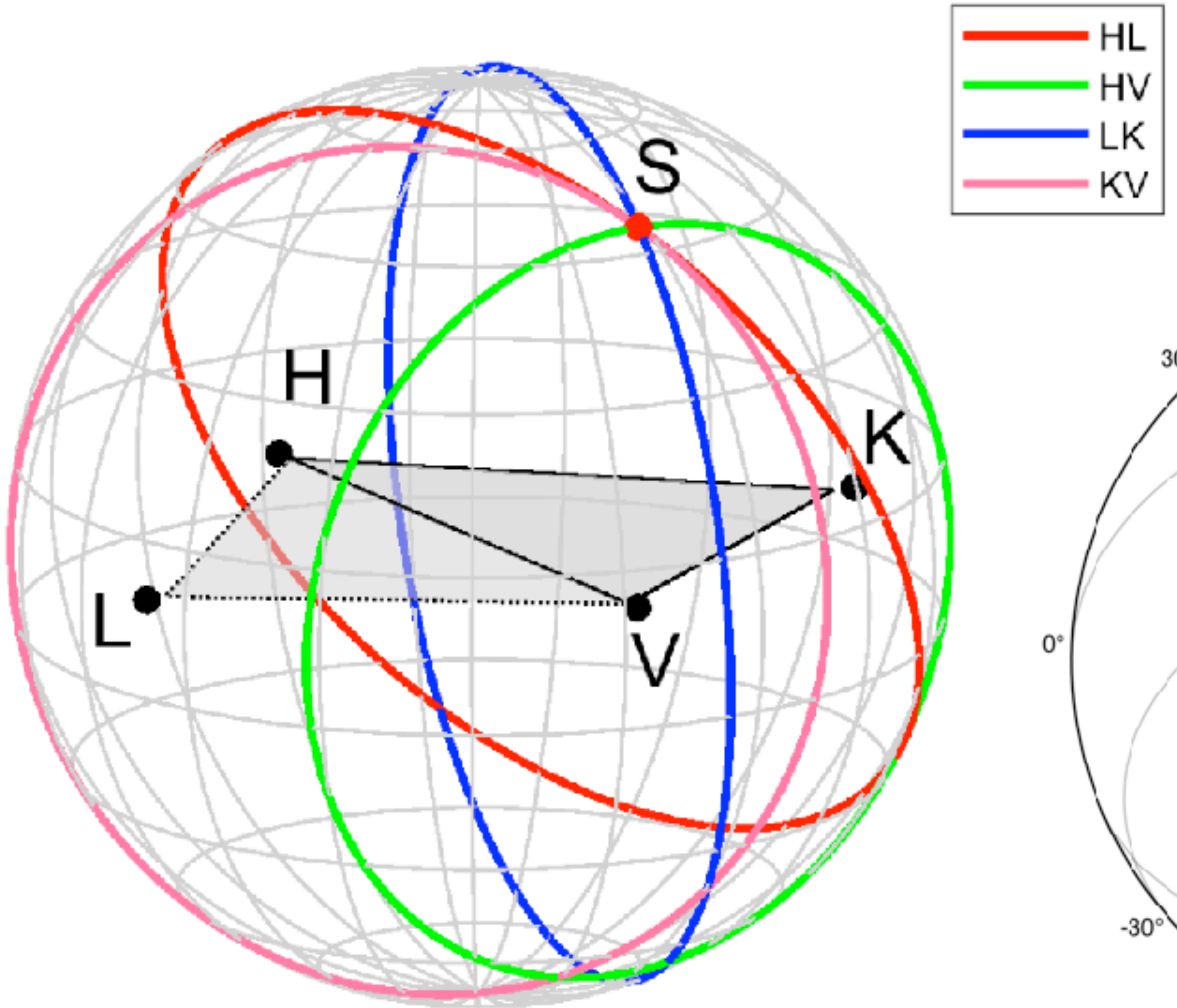
- Individual GW detectors are omnidirectional: poor localisation! Sensitivity depends on location, polarisation and frequency
- Simultaneously operating observatories allow for triangulation via arrival time differences

[Sathyaprakash&Schutz, LRR]

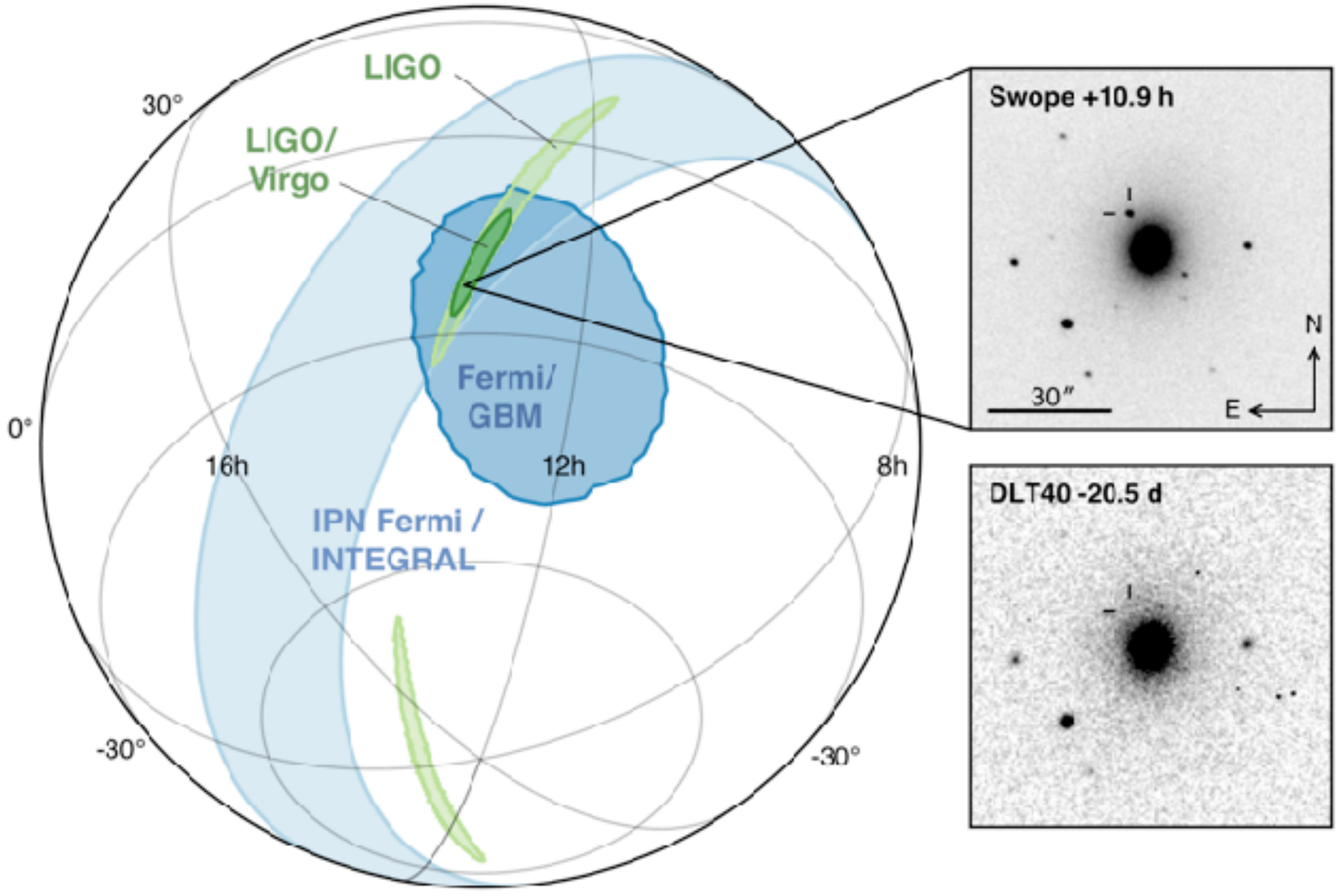
RMS antenna response



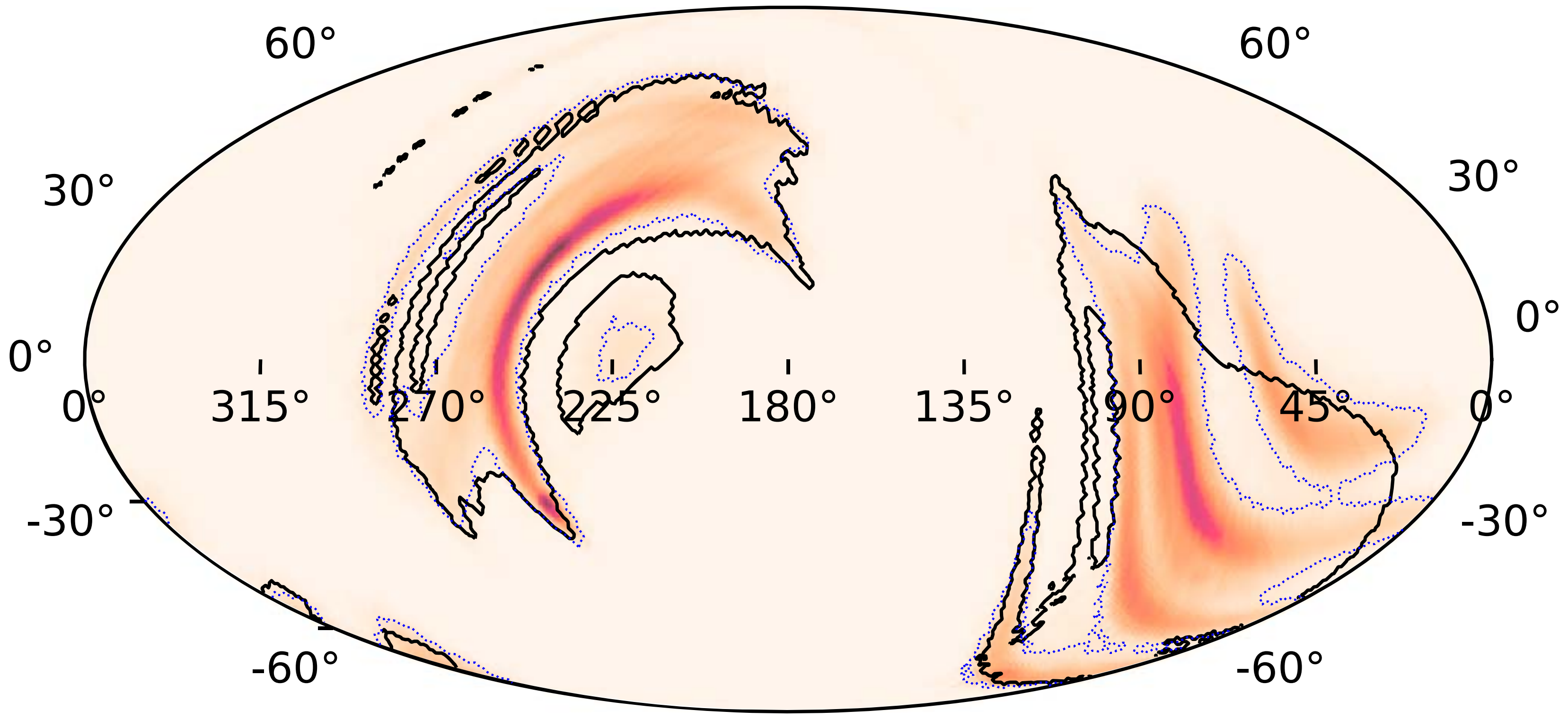




GW170817



GW190425



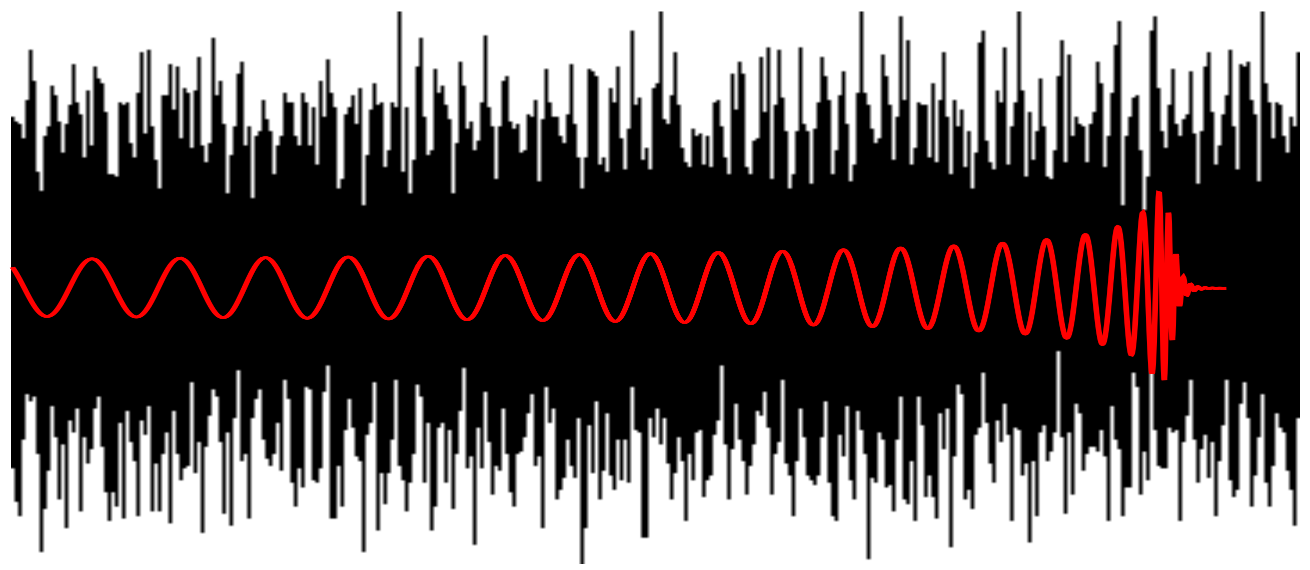
90% CI ~ 8300 deg^2

[LVK Observing Scenarios & LVK discovery papers]

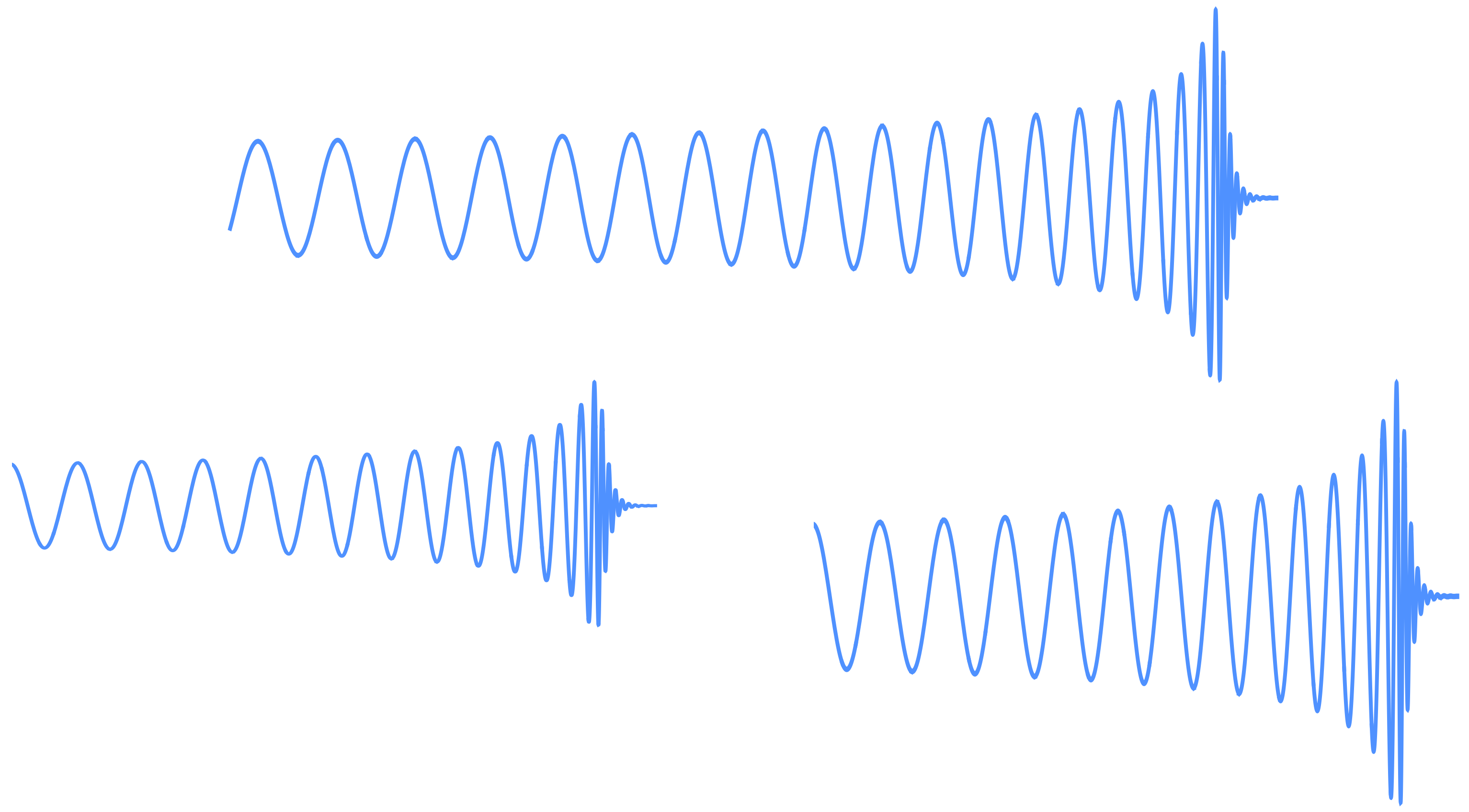
MODELLED SEARCHES

- Optimal detection strategy for a **modelled signal** in stationary Gaussian noise = **matched filtering**

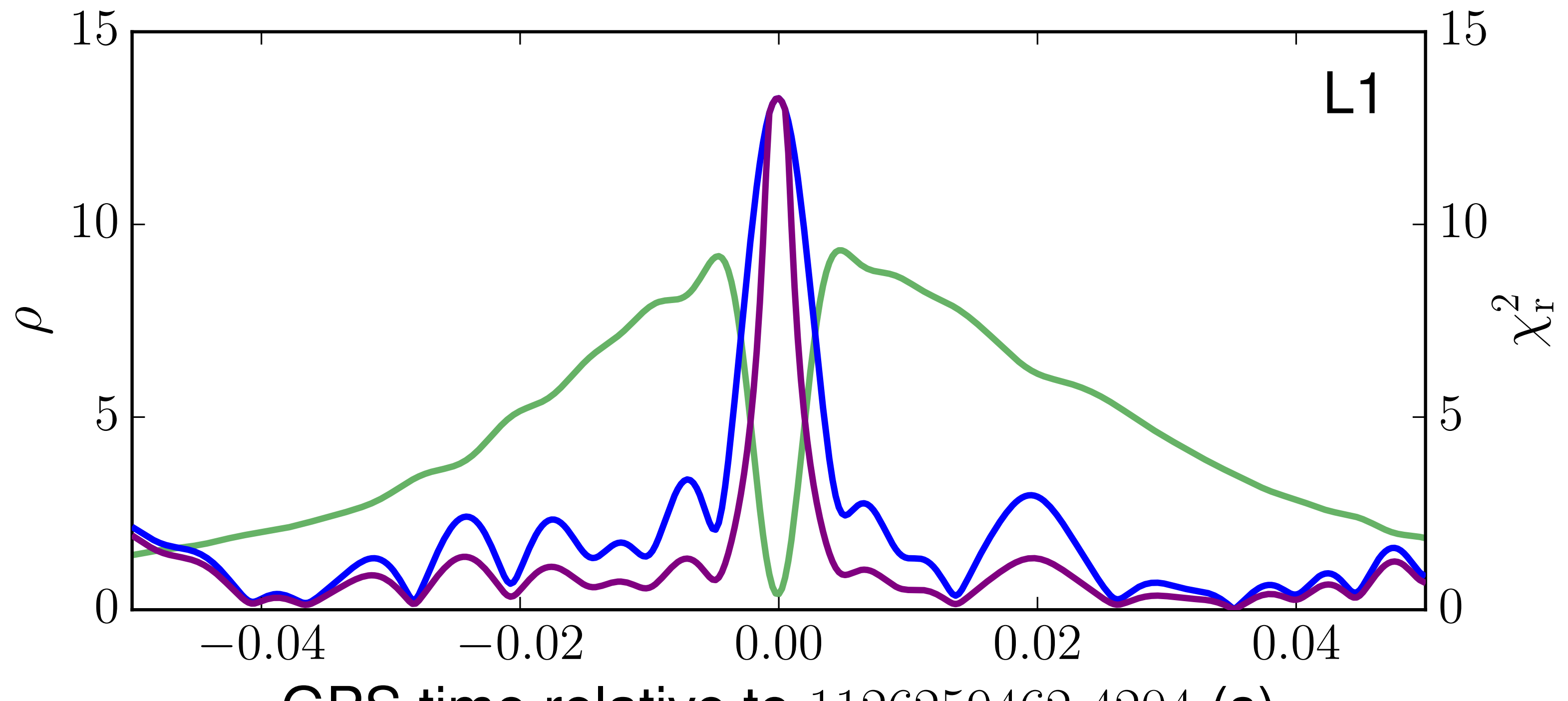
strain data d



template waveforms h



SNR time series

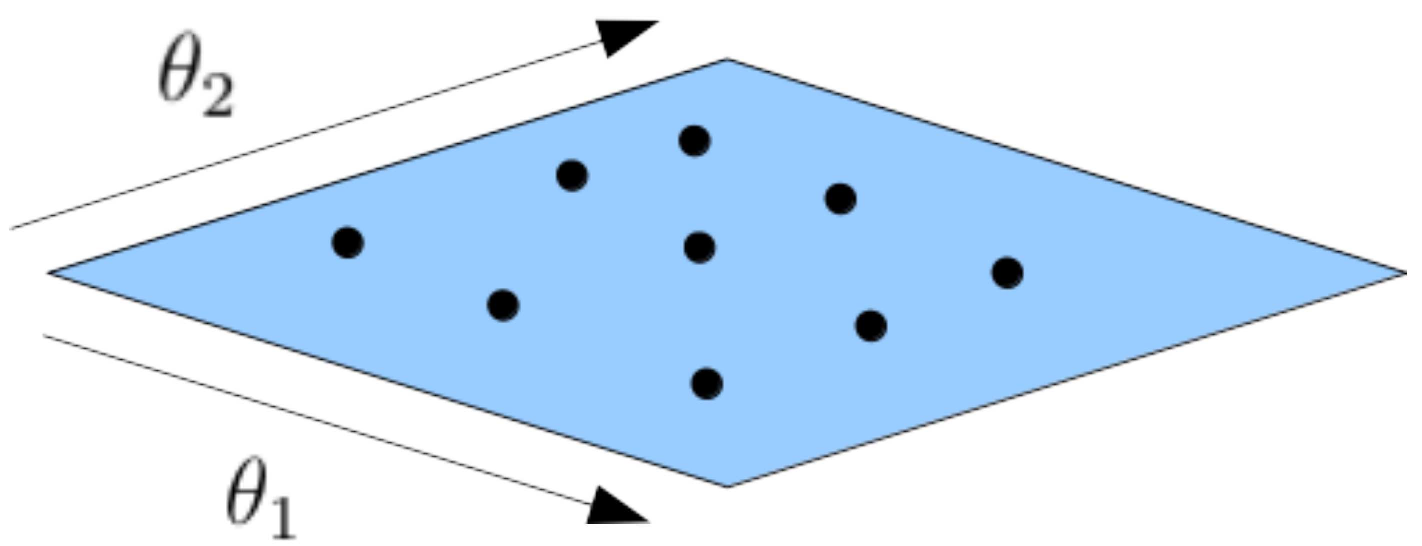


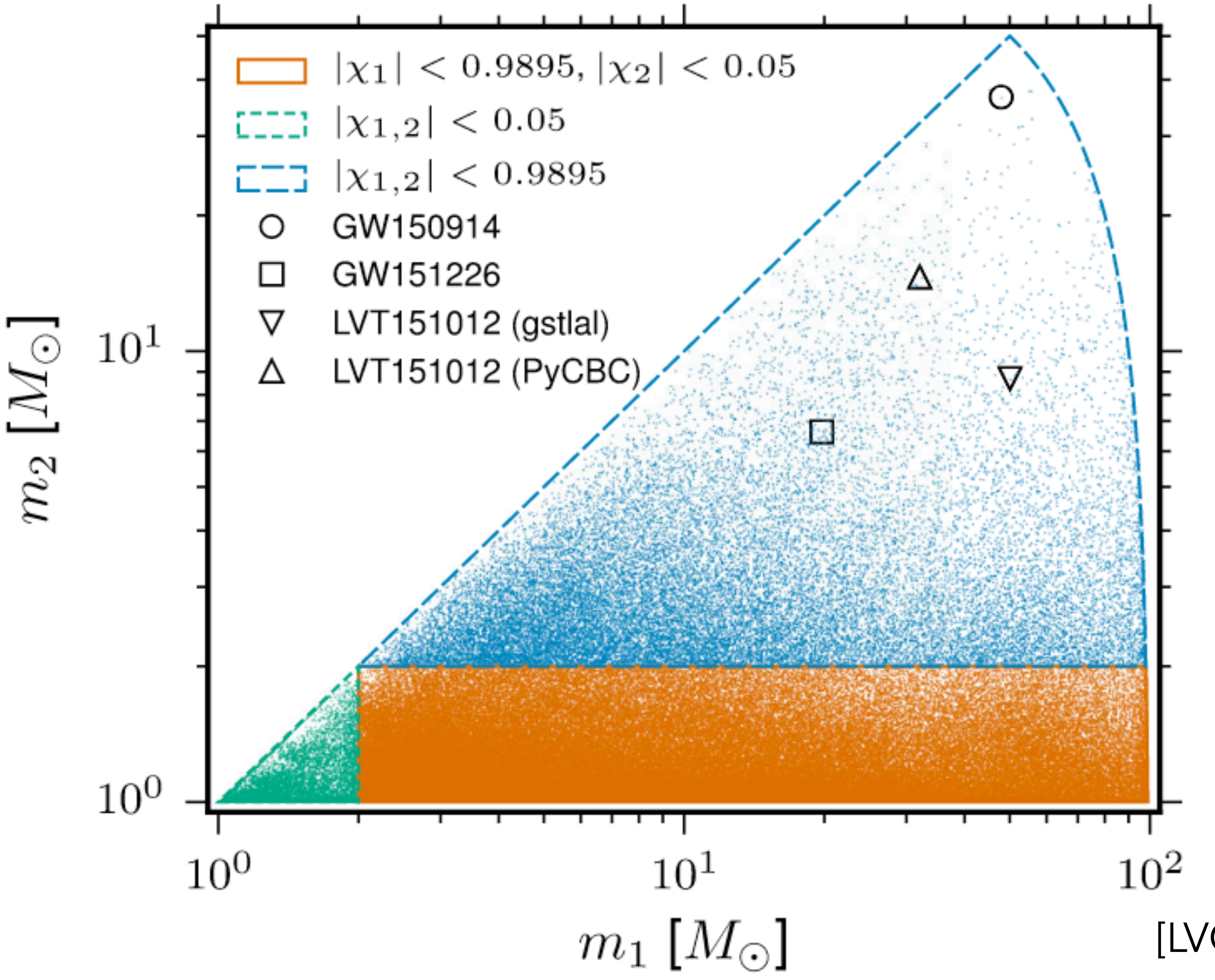
- **Optimal filter:** $$\rho = \frac{\langle d|h\rangle}{\sqrt{\langle h|h\rangle}}$$ $$\langle a|b\rangle = 4\int_0^\infty df\,\frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}$$

signal-to-noise ratio (SNR)

TEMPLATE BANKS

- Large collection of theoretical waveforms templates:
- Construction: hybrid method
  - Geometric lattice for known metric (green area)
    - Metric = mismatch between neighbouring templates $1 - \langle \hat{h}(\theta) \,|\, \hat{h}(\theta + \Delta\theta) \rangle = g_{ij} \Delta\theta_i \Delta\theta_j$
  - Stochastic placement
- What's included?
  - Component masses
  - Aligned-spins
  - Quadrupolar mode



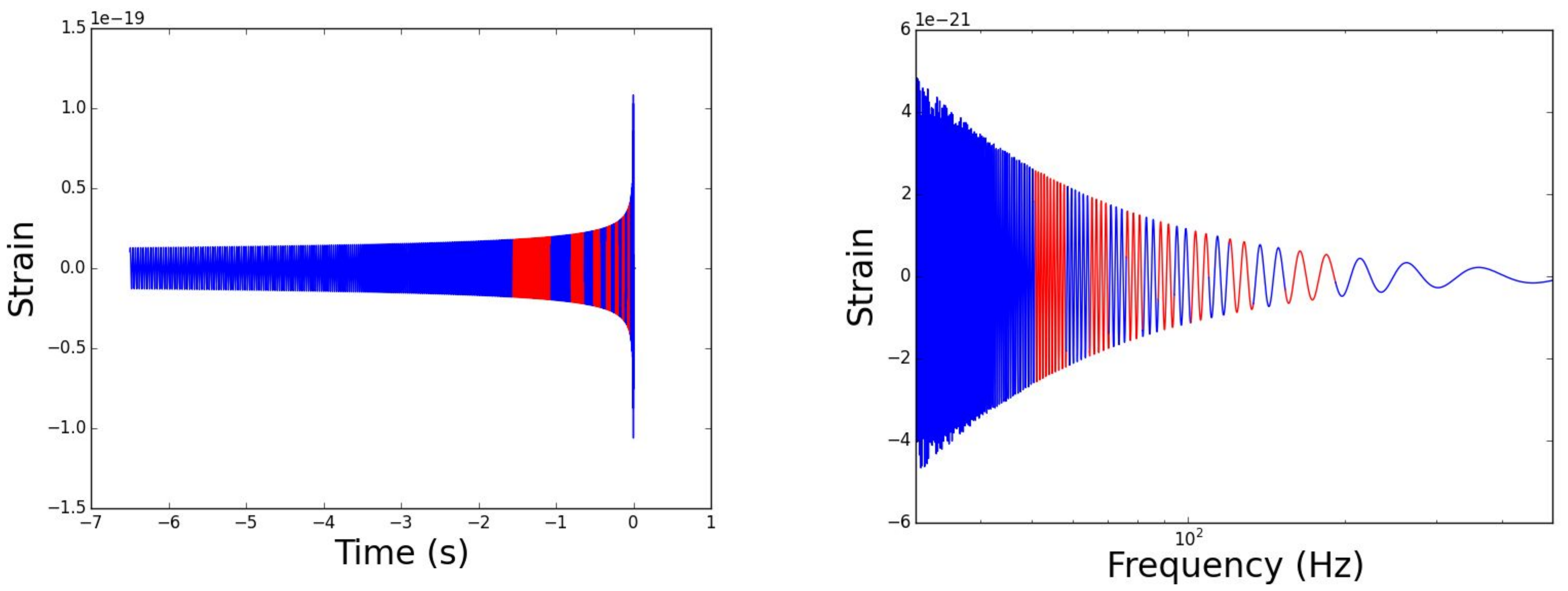


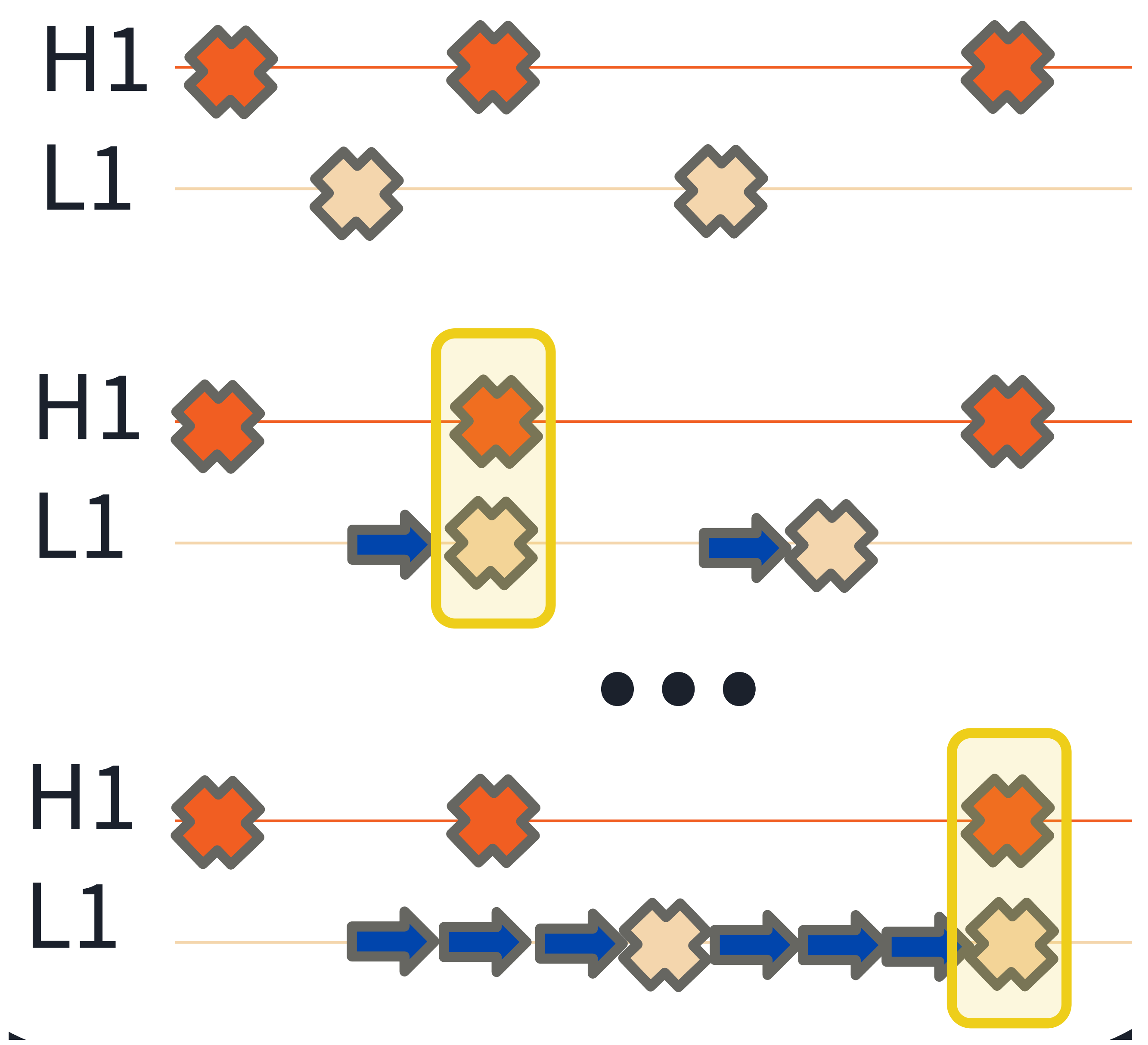
[LVC, O1]

EVENT SELECTION

- Apply any data excursions
- Filter each data stream individually
- Apply threshold SNR cut and cluster to generate triggers
- Perform χ^2-test to check signal consistency
- Check time & parameter coincidence between different IFOs
- Apply data quality vetoes
- Surviving triggers form GW candidates
- Use **time shifts** to calculate the **false-alarm-rate (FAR)** of coincident triggers. Resulting triggers form the background, which is used to determine the significance of the foreground candidates.

Example: SNR accumulation as signal-consistency test



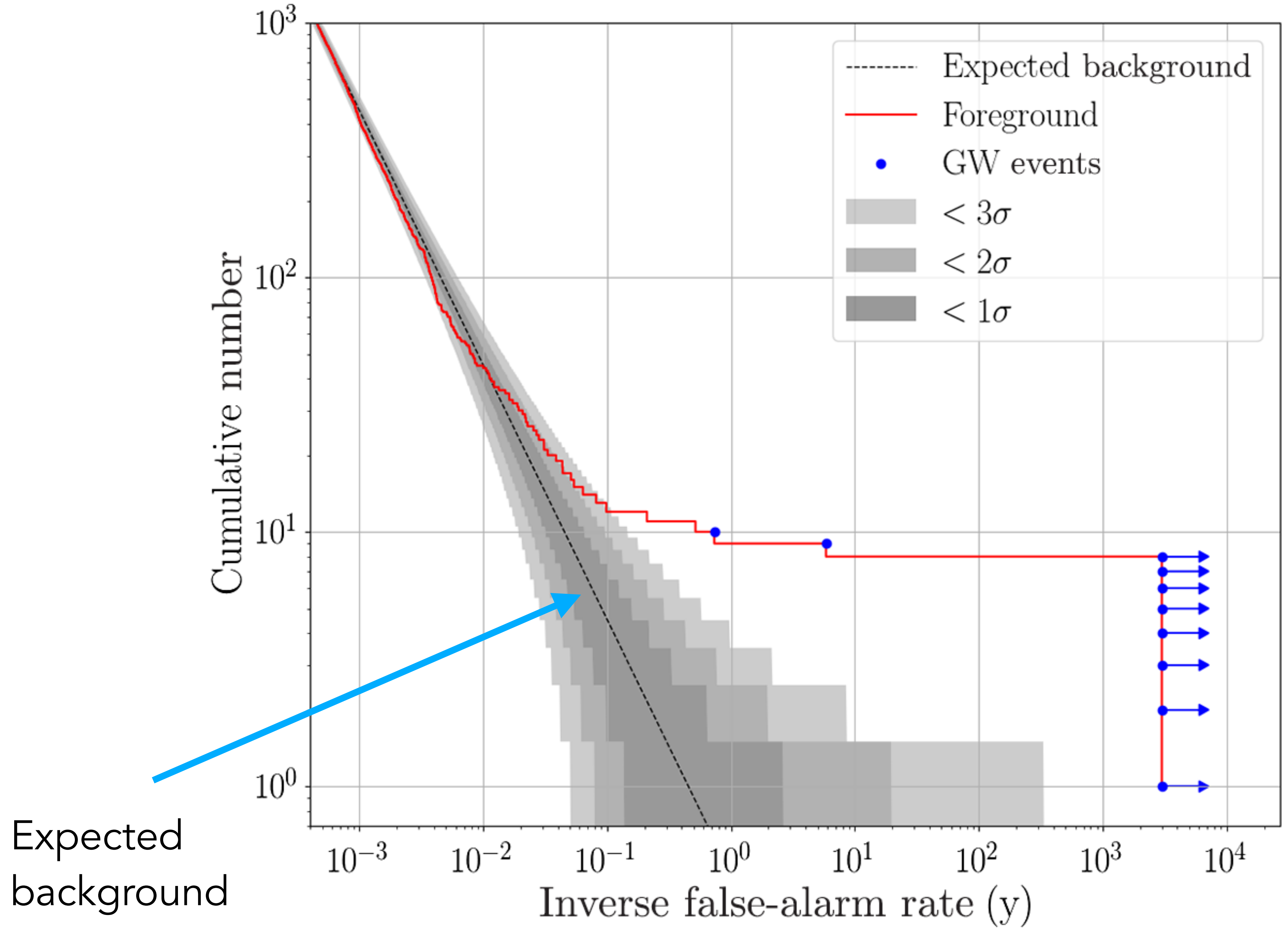


Credit: A. Nitz

SIGNIFICANCE

- **FAR** (caution: varies!): e.g. O2 1/30 days (expect ~2 false alarms per month)

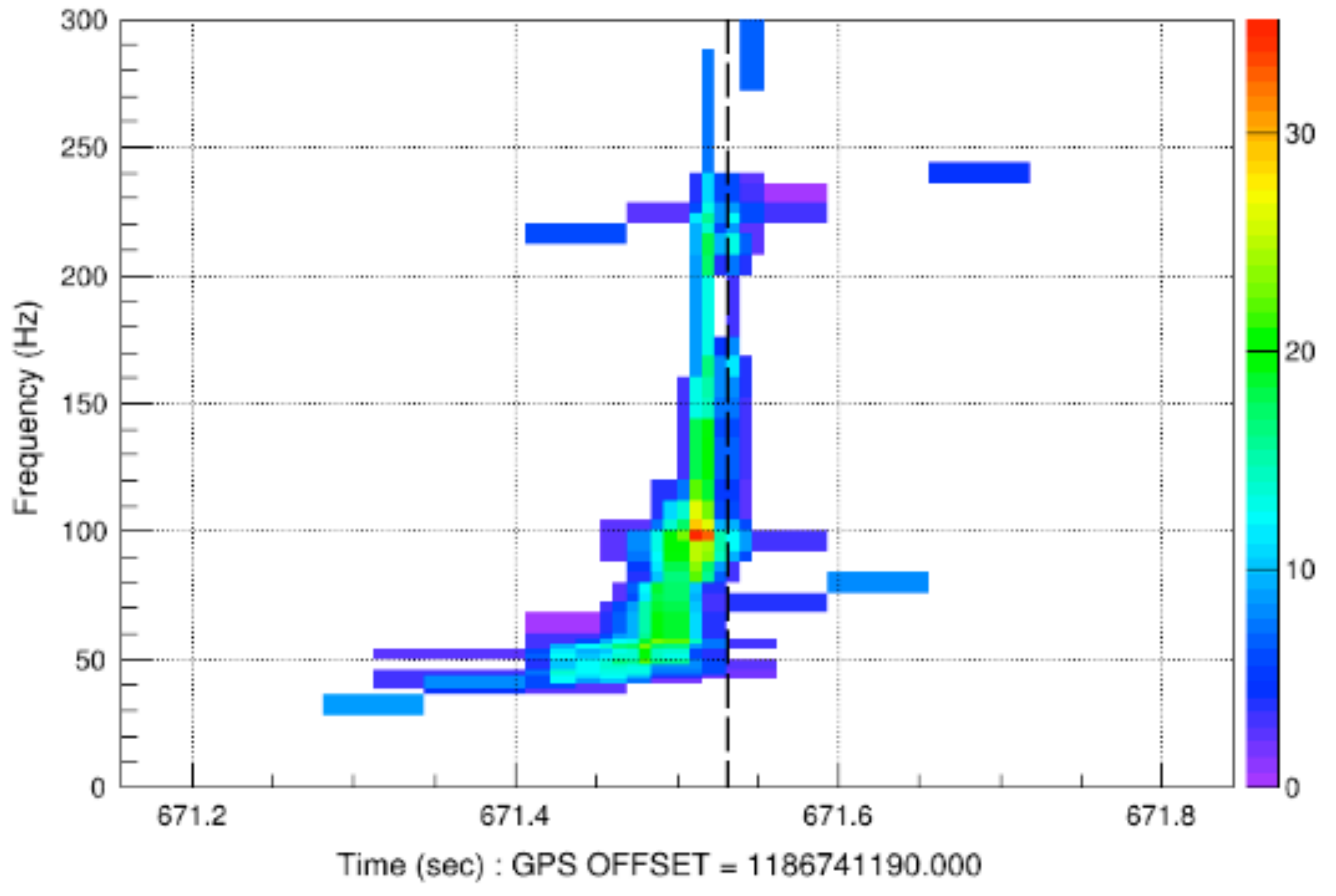
- **Astrophysical probability**: assumes 4 categories A_i (terrestrial, BBH, BNS, NSBH) each described by a Poisson process (inhomogeneous Poisson mixture model) with mean Λ_i [Farr+, 2013]
  - Ranking statistic distribution: $p(x\,|\,A_i, \{\theta\})$
  - $\Lambda_i = \langle VT \rangle_{\{\theta\}} \times R_i$
    - Observable spacetime volume (selection effects!)
  - Models are marginalised over the counts with the ranking statistic distribution fixed at the value of the ranking statistic of the candidate



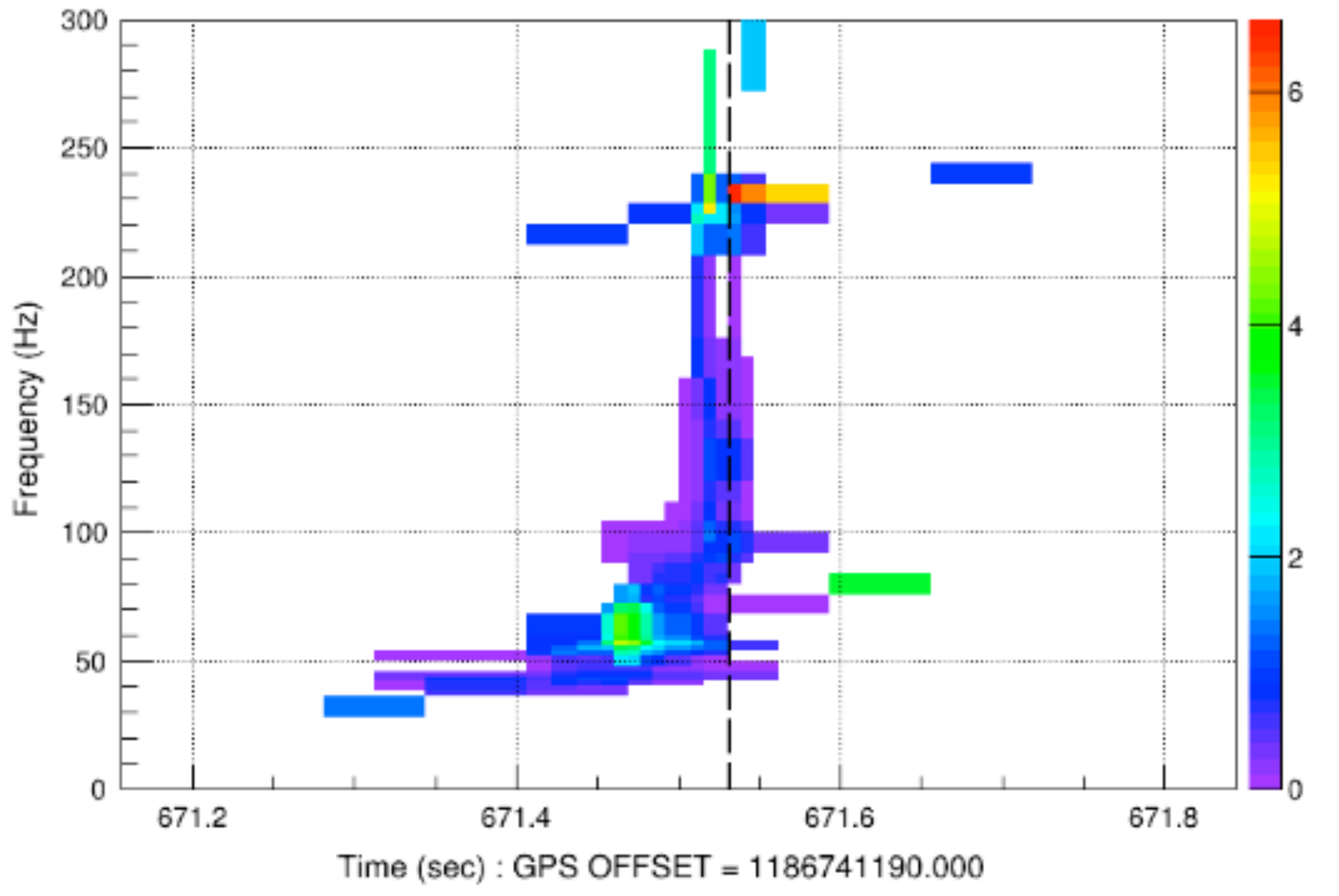
[LVK, GWTC-1]

UNMODELLED SEARCHES

- Search that identifies **coincident excess power** in the time-frequency representation of the strain data
    - Identifies events that are coherent in multiple detectors and reconstructs the source sky location and signal waveforms by using the constrained maximum likelihood method
    - Does not rely on waveform models
    - Sensitive to a wide range of short-duration transient signals (“bursts”)
    - Weak assumption of “chirpyness” of the signal
- Detection statistic: coherent energy constructed via cross-correlation $E_c \propto \rho_c$



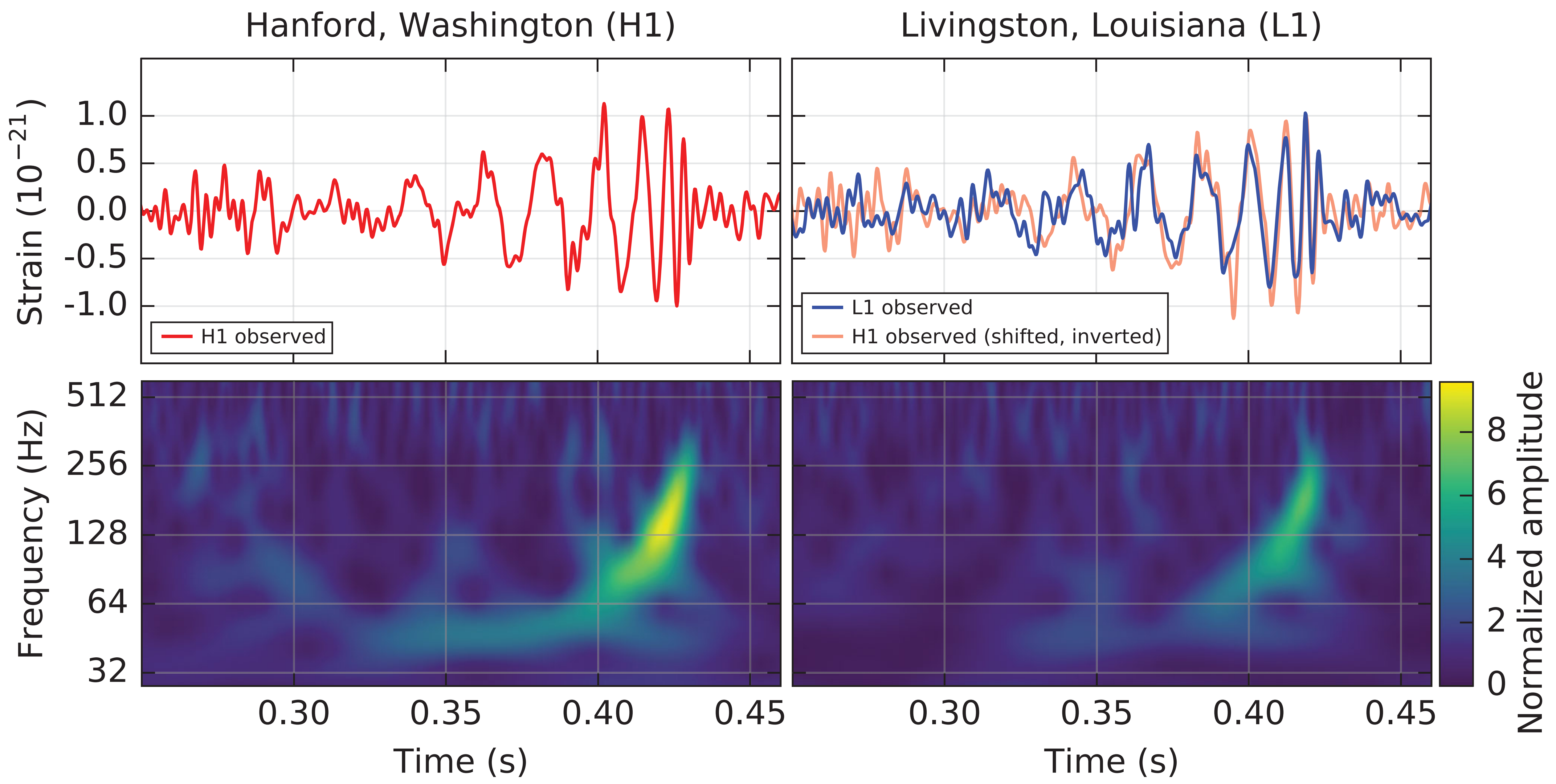
(c)

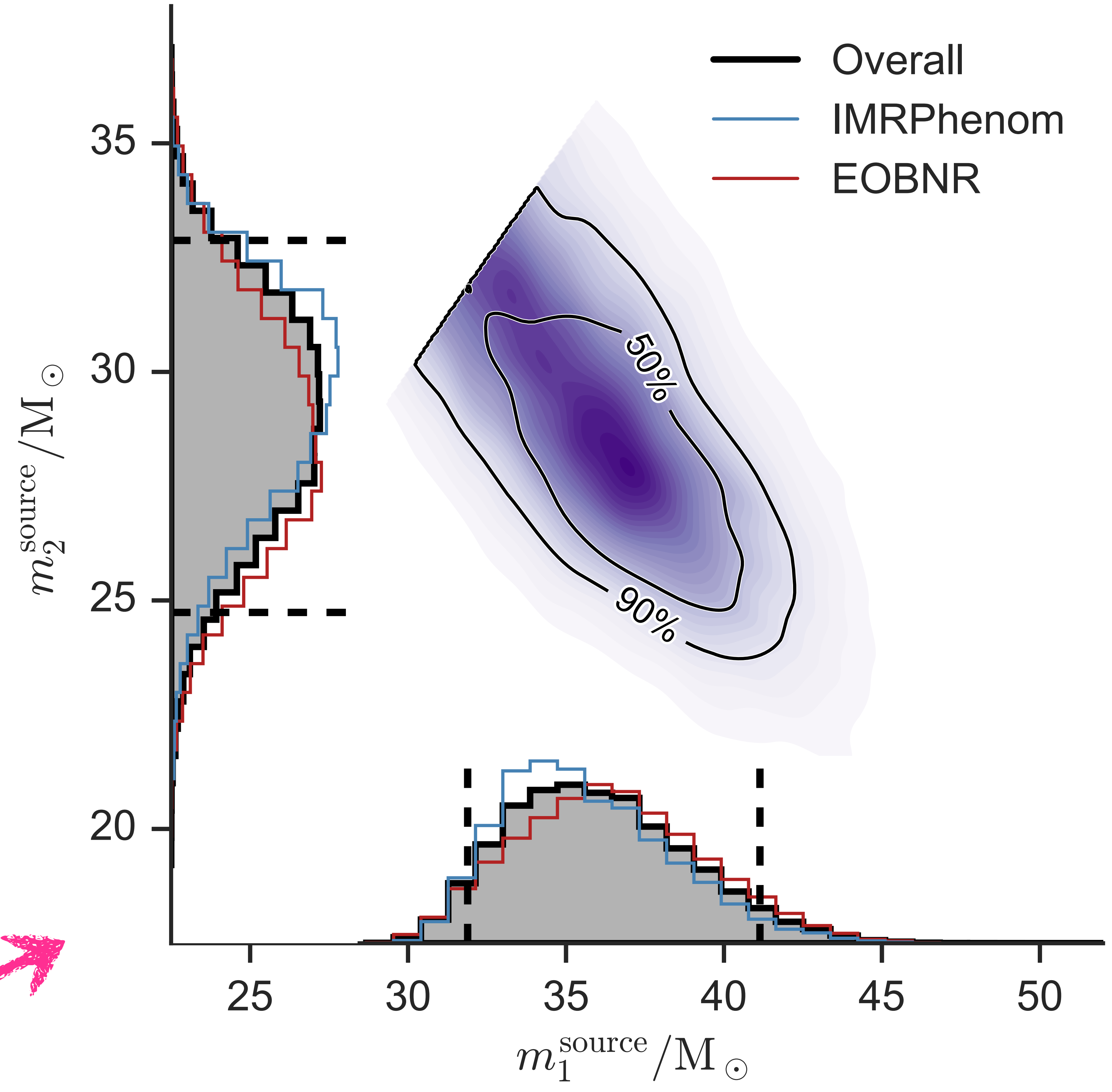


[Salemi+, 2019]

SOURCE CHARACTERISATION

Calibrated (raw-ish) strain data → **Astrophysical parameter distributions**

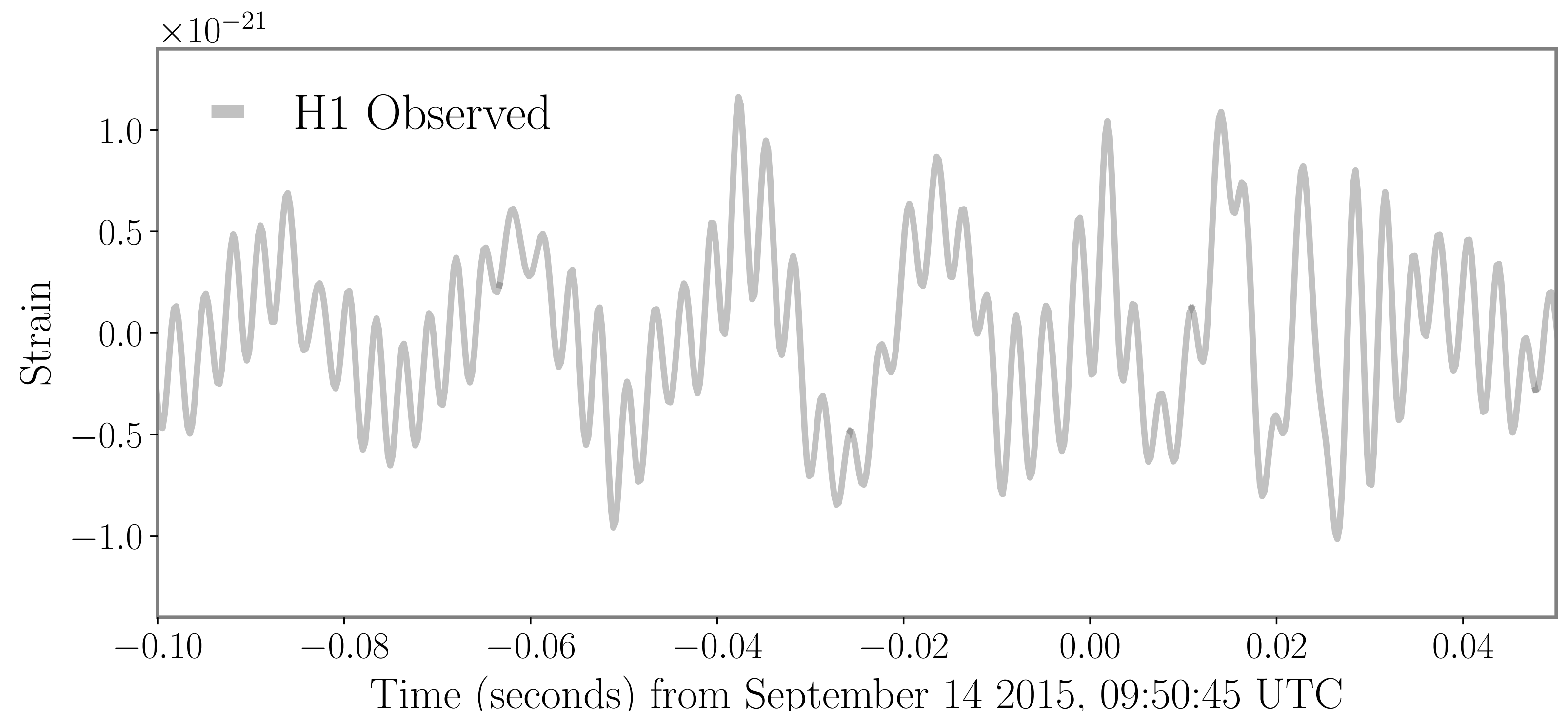




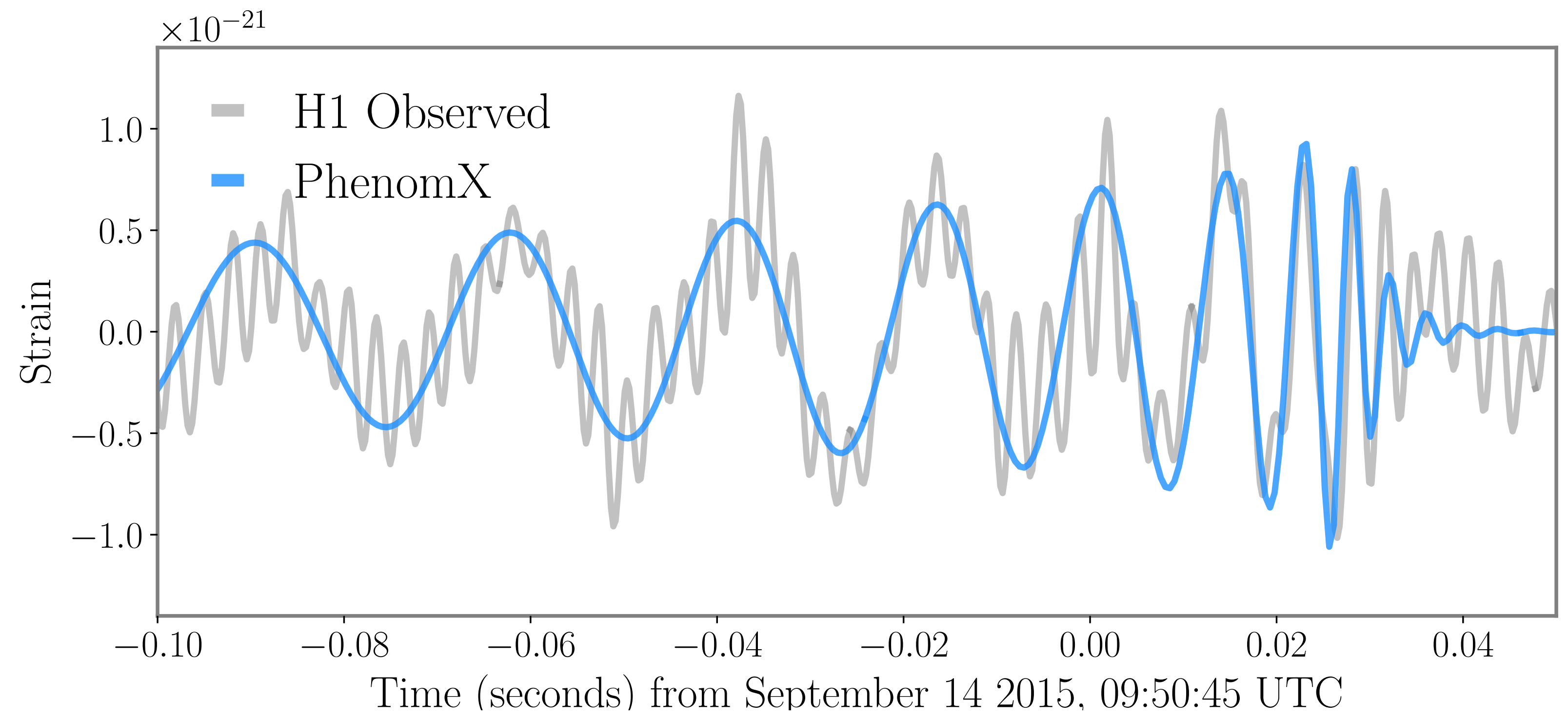
[LVC, GW150914 discovery papers]

MEASURING BINARY PARAMETERS

- GW signal encodes the astrophysical information: masses, spins, tides, location, orientation of the orbit



- Use **Bayesian inference** to infer the source parameters θ:

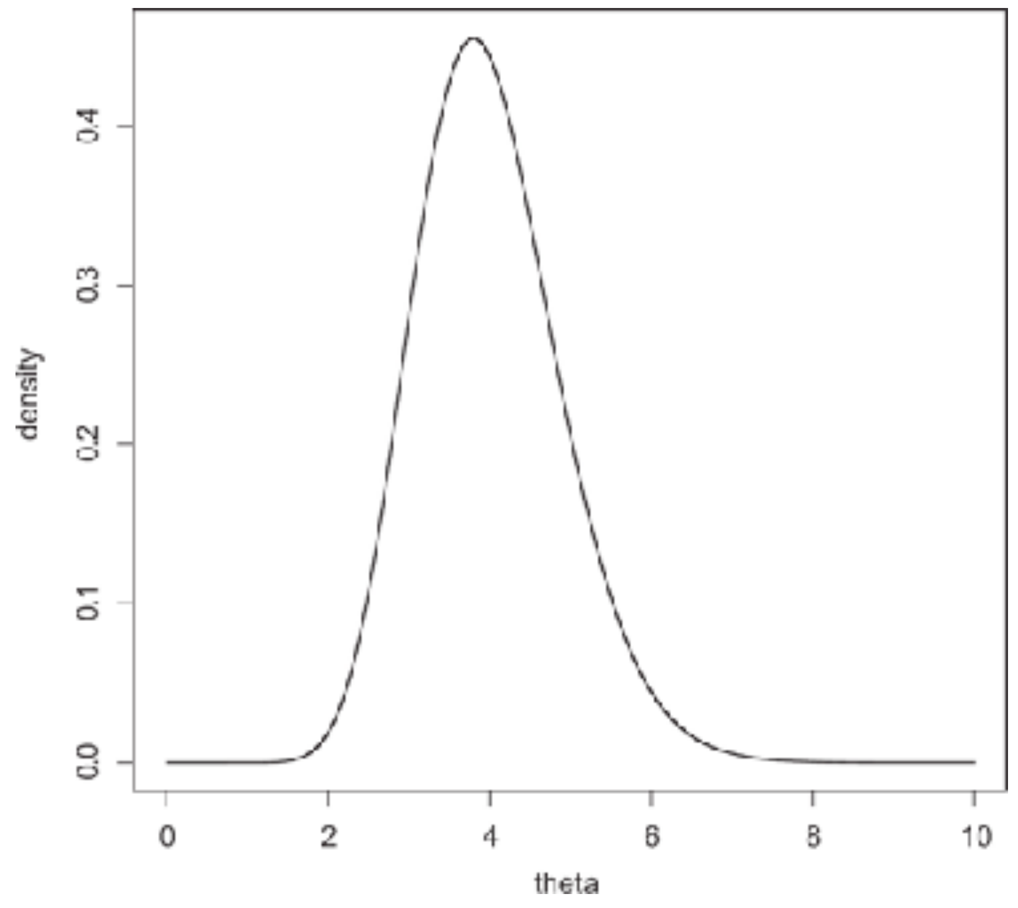


Likelihood Priors

Posterior Probability

$$p(\theta \mid d) = \frac{\mathcal{L}(d \mid \theta)\pi(\theta)}{\mathcal{Z}}$$

Evidence

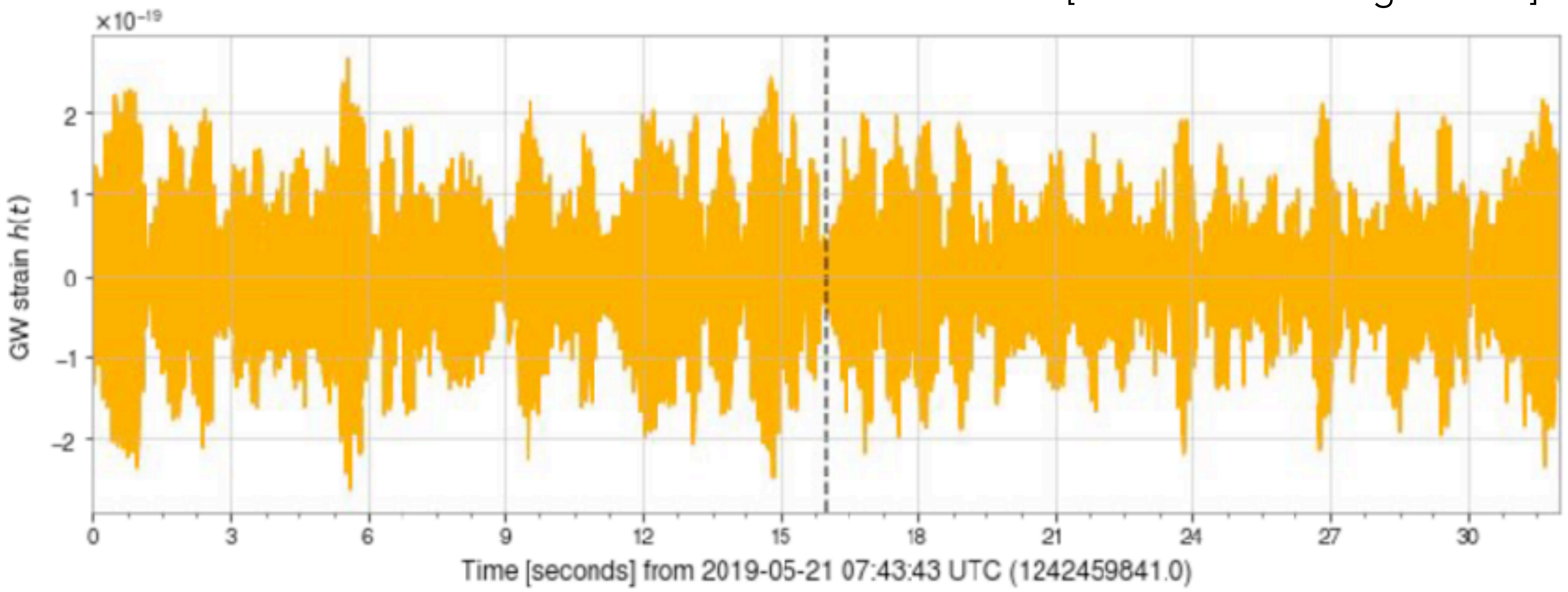


Slide credit: G. Pratten

THE DATA

- For an incident signal, a GW detector records the following data:

[Credit: A. Helmling-Cornell]



$$d = R(h) + n$$

data

detector response to a GW

noise

- Real-valued detector response to a GW: $R(h) = F_+(\alpha, \delta, \psi) h_+ + F_\times(\alpha, \delta, \psi) h_\times$

THE NOISE

- Three key assumptions:
  - **Gaussian** with zero mean and known variance

noise correlation matrix

$$p(n) \propto \exp\left(-\frac{1}{2} \sum_{i,j} n_i C_{ij}^{-1} n_j \right)$$

  - **Stationary**

$$C_{ij} = \frac{1}{2} S_n(f_i) \delta_{ij}$$

power spectral density

  - **Independent** between frequency bins and detectors

LIKELIHOOD

- We are free to choose the likelihood
- In GW astronomy we assume a likelihood associated with **stationary Gaussian noise** that has zero mean and a known variance:

waveform template

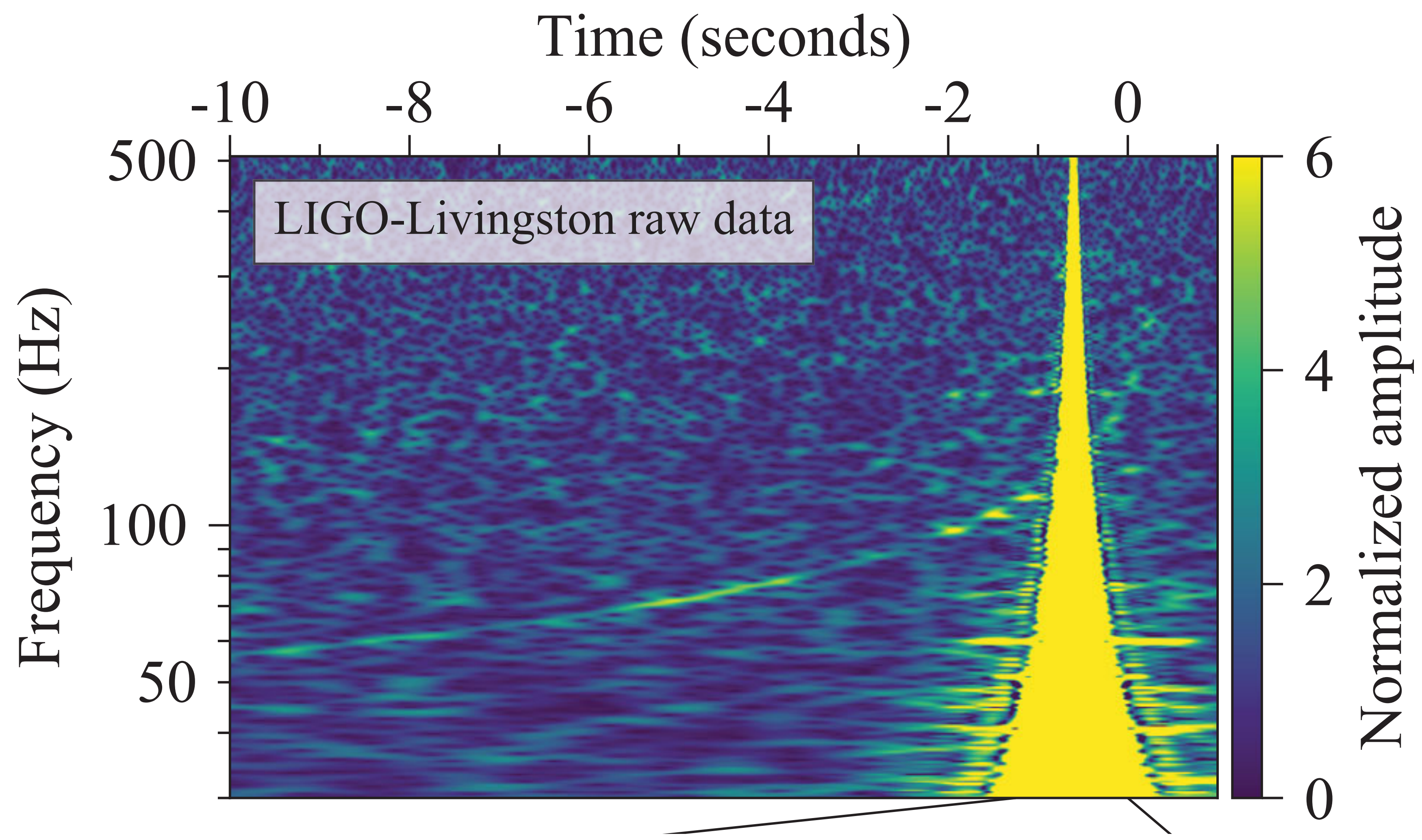
$$\mathcal{L}(d|\theta, H, S_n(f)) = \exp\left(\sum_i -\frac{2|\tilde{h}_i(\theta) - \tilde{d}_i|^2}{T S_n(f_i)} - \frac{1}{2}\log\left(\pi T S_n(f_i)/2\right)\right)$$

- Likelihood compares theoretical models against the data
- For a network of GW detectors:

$$\mathcal{L}(d_{\mathrm{NW}}|\theta, H, S_{n\mathrm{NW}}(f)) = \prod_{i\in\mathrm{NW}} \mathcal{L}(d_i|\theta, H, S_{ni}(f))$$

STATIONARY & GAUSSIAN?

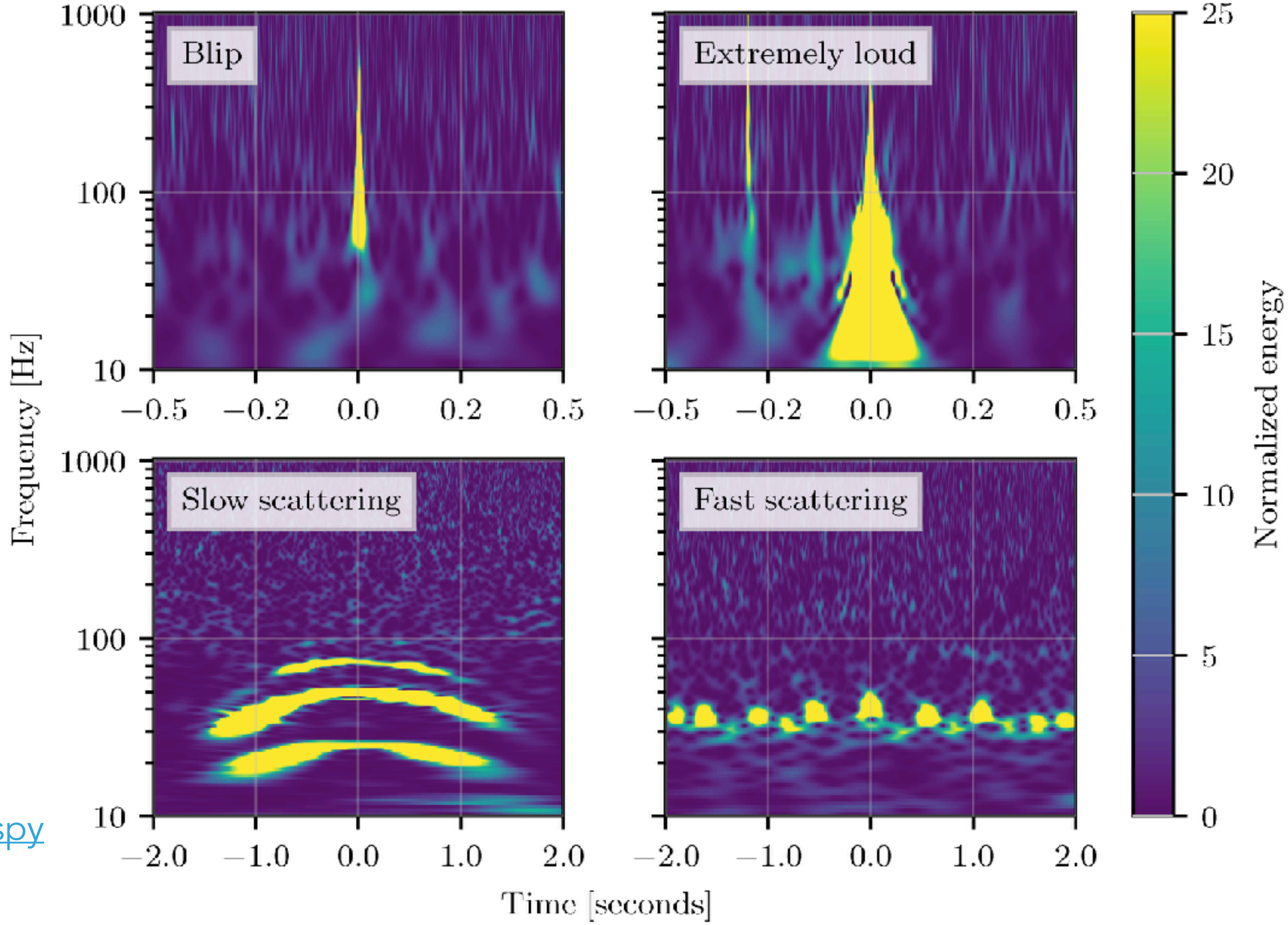
- Data cleaning
- Glitch removal/mitigation



Citizen science: Gravity Spy

https://www.zooniverse.org/projects/zooniverse/gravity-spy

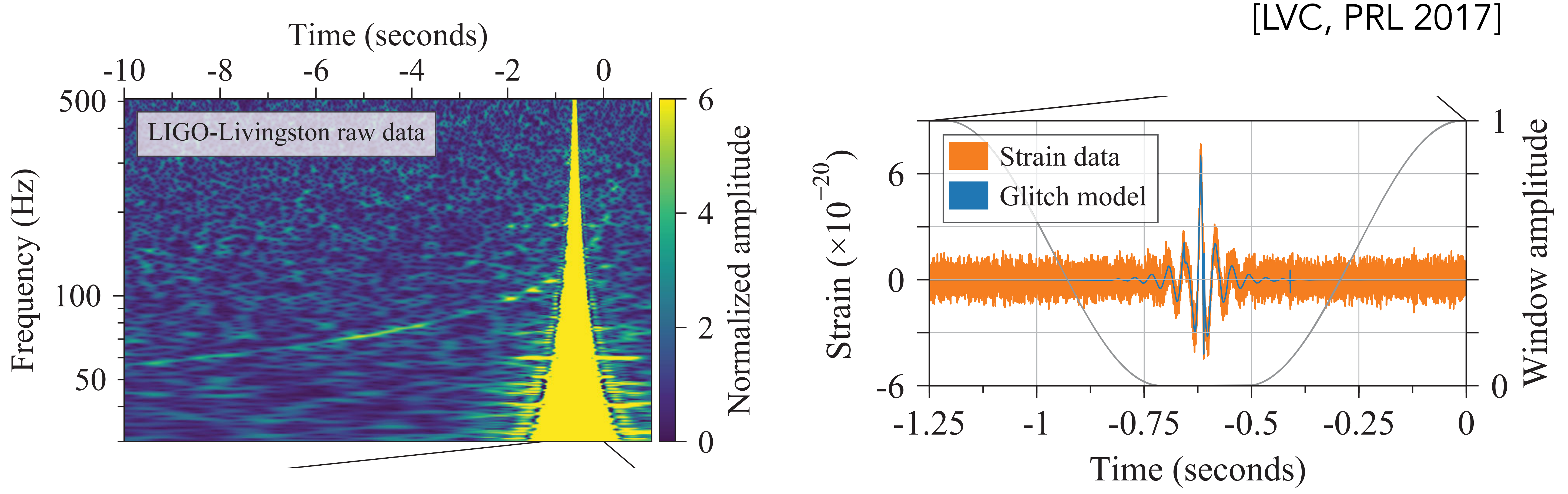
Some example **"glitches"**



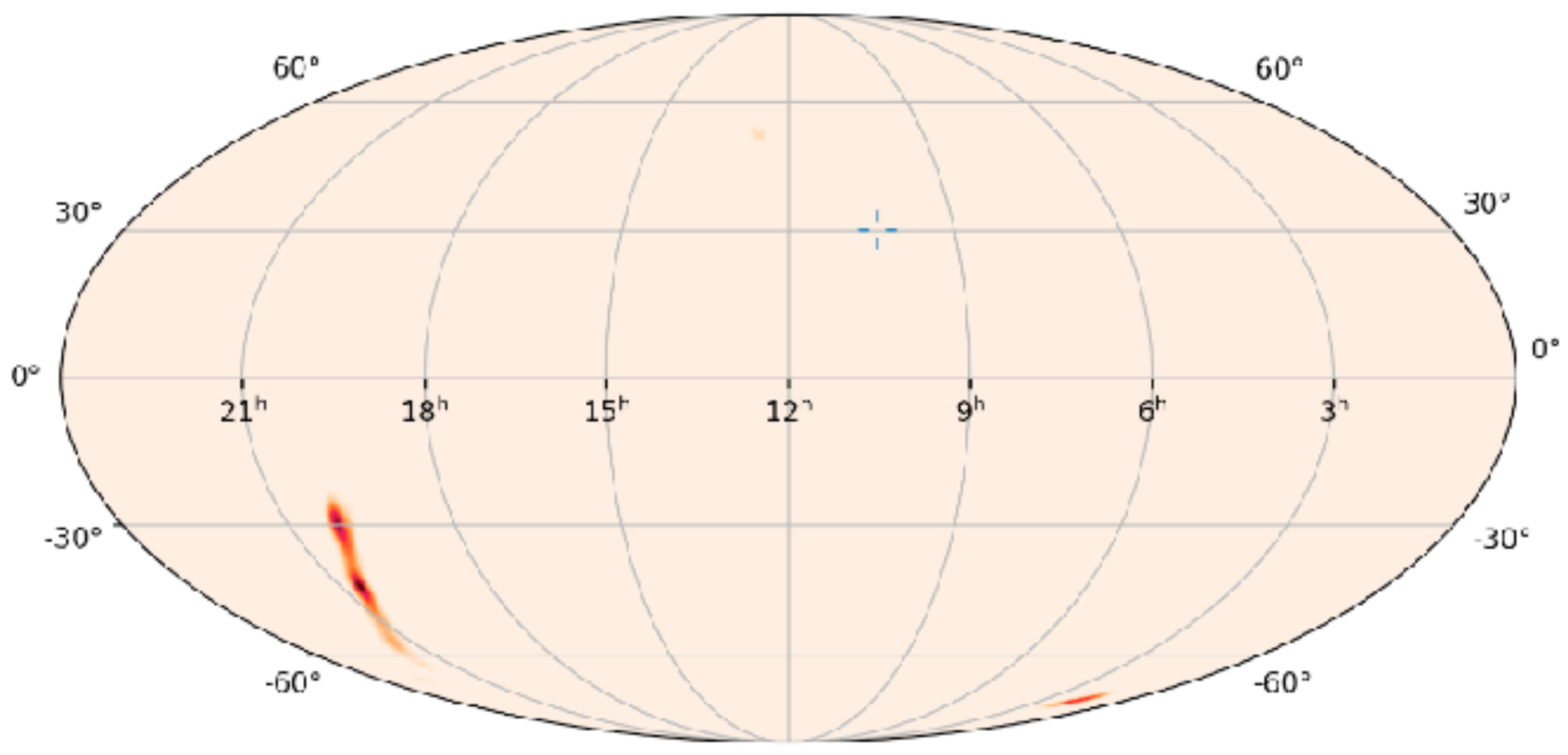
GAUSSIANITY

- Real detector noise often contains **non-Gaussianities** ("glitches")
    - O3: 24% of GW candidates near glitches
    - We can modify the likelihood or remove them from the data
- Prominent example: GW170817
- Noise transients impact the measurement:
    - Systematic errors in sky location
    - Systematic biases in source parameters

[LVC, PRL 2017]



Example: GW170817 overlapped with a blip glitch in LIGO-Livingston

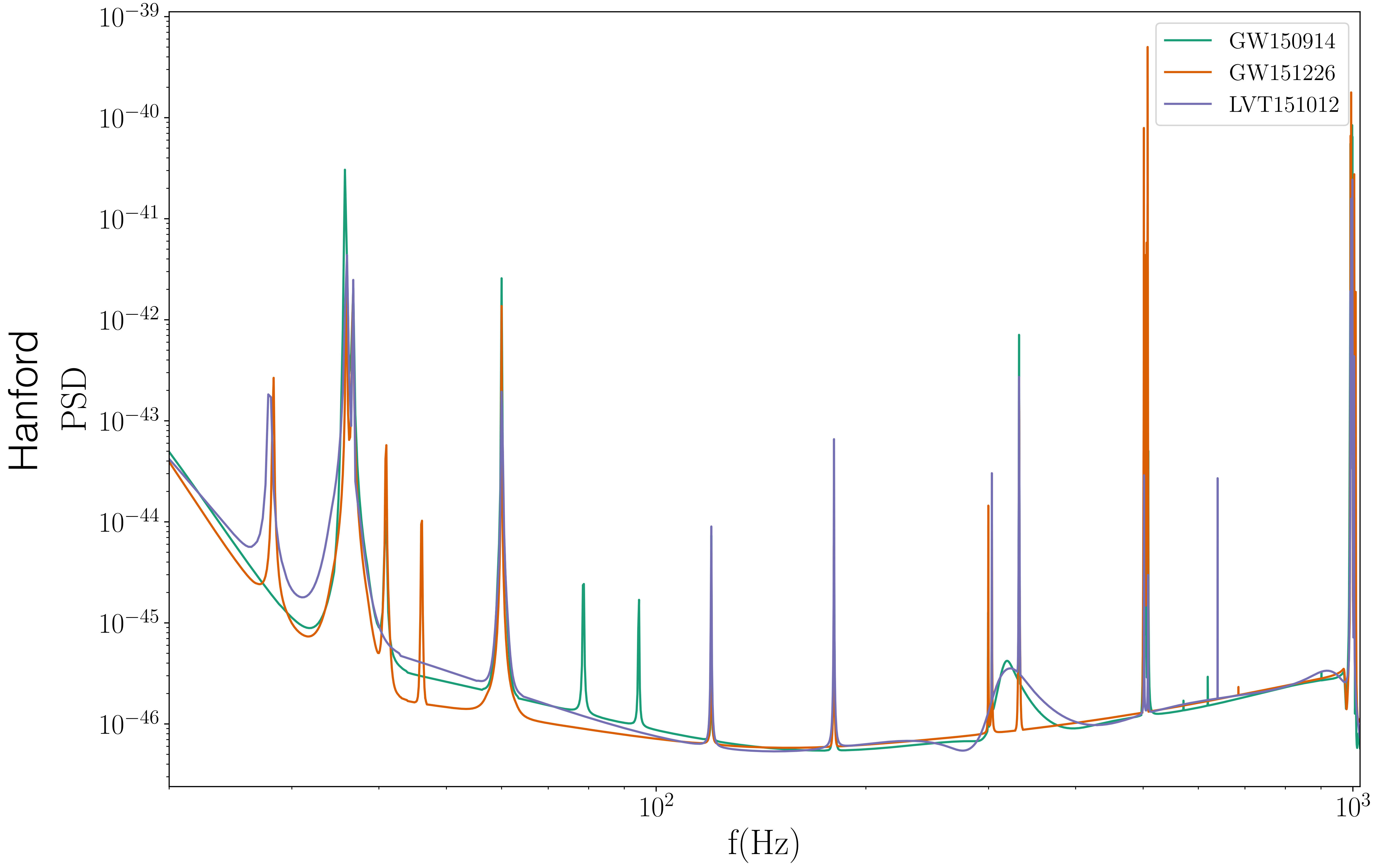


(a) Sky localisation of a GW150914-like event injected at $t_0 + 30\,\text{ms}$ relative to the blip glitch central time t_0. The 90% credible area is 137 deg^2.

[Macas+, 2022]

STATIONARITY

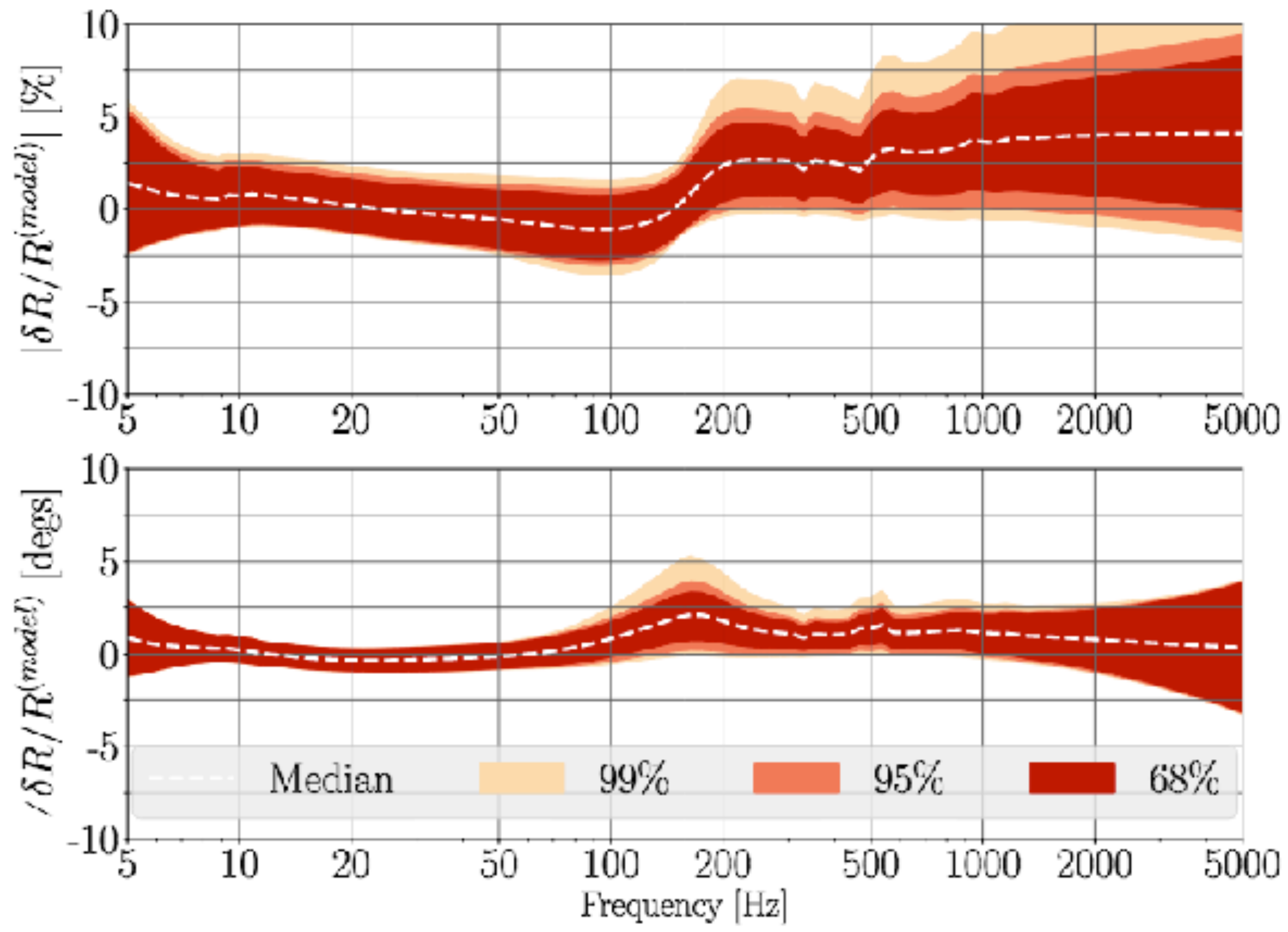
- The noise floor in GW detectors varies over time
  - $S_n(f)$ changes between events
  - Compute the PSD for every event to take PSD drift into account
- Note: Uncertainty in the PSD estimation can also be taken into account via marginalisation by introducing extra parameters into the noise model

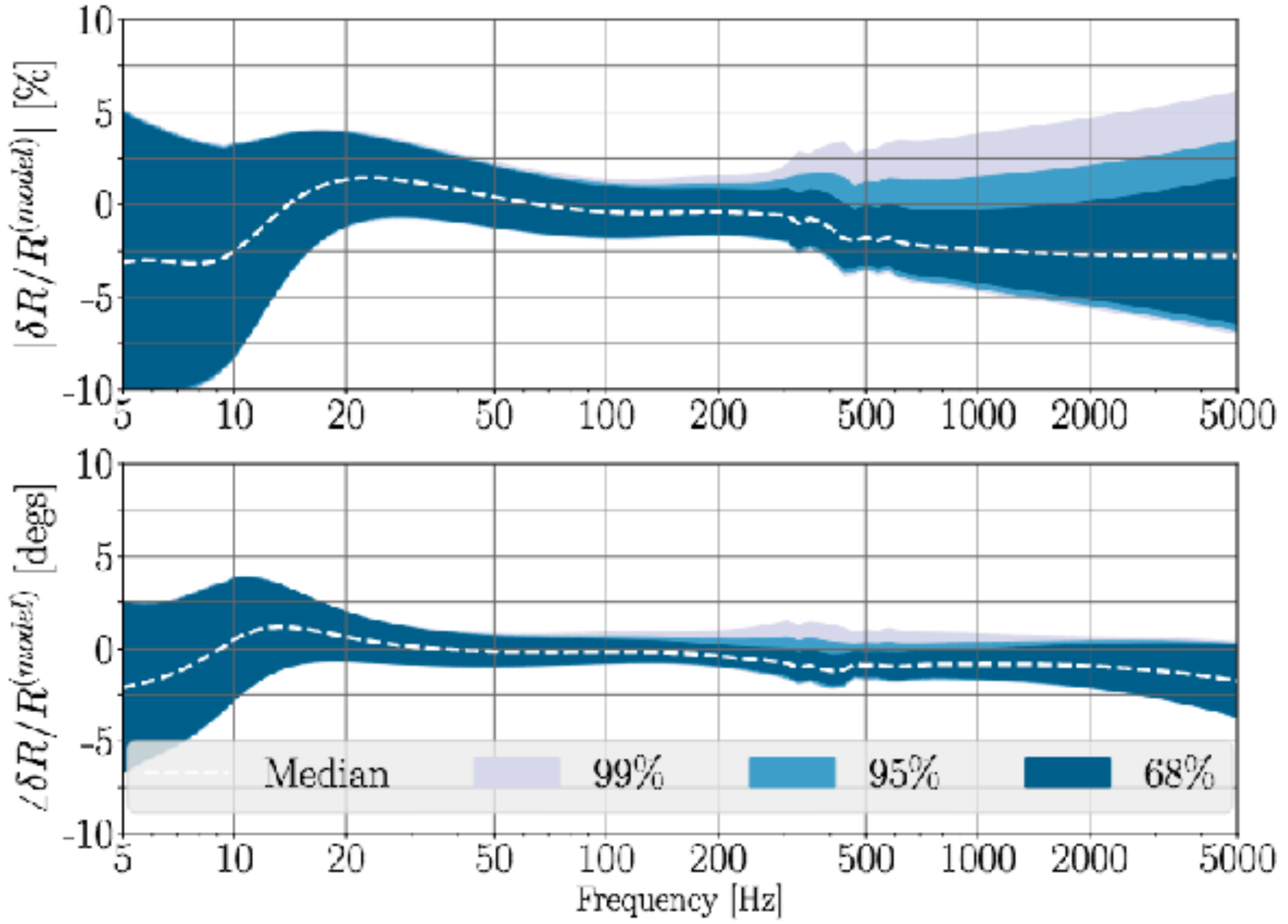


Credit: K. Chatziioannou

CALIBRATION

- Calibration quantifies the detector's response to incident GWs
  - Miscalibration results in biased strain data!
- Parameterised model for calibration uncertainty
  - Marginalised over during parameter estimation
- Calibration uncertainties are not the limiting factor in current GW observations
  - Statistical uncertainty from detector noise dominates





[Cahillane+, 2017]]

THE SIGNAL MODEL

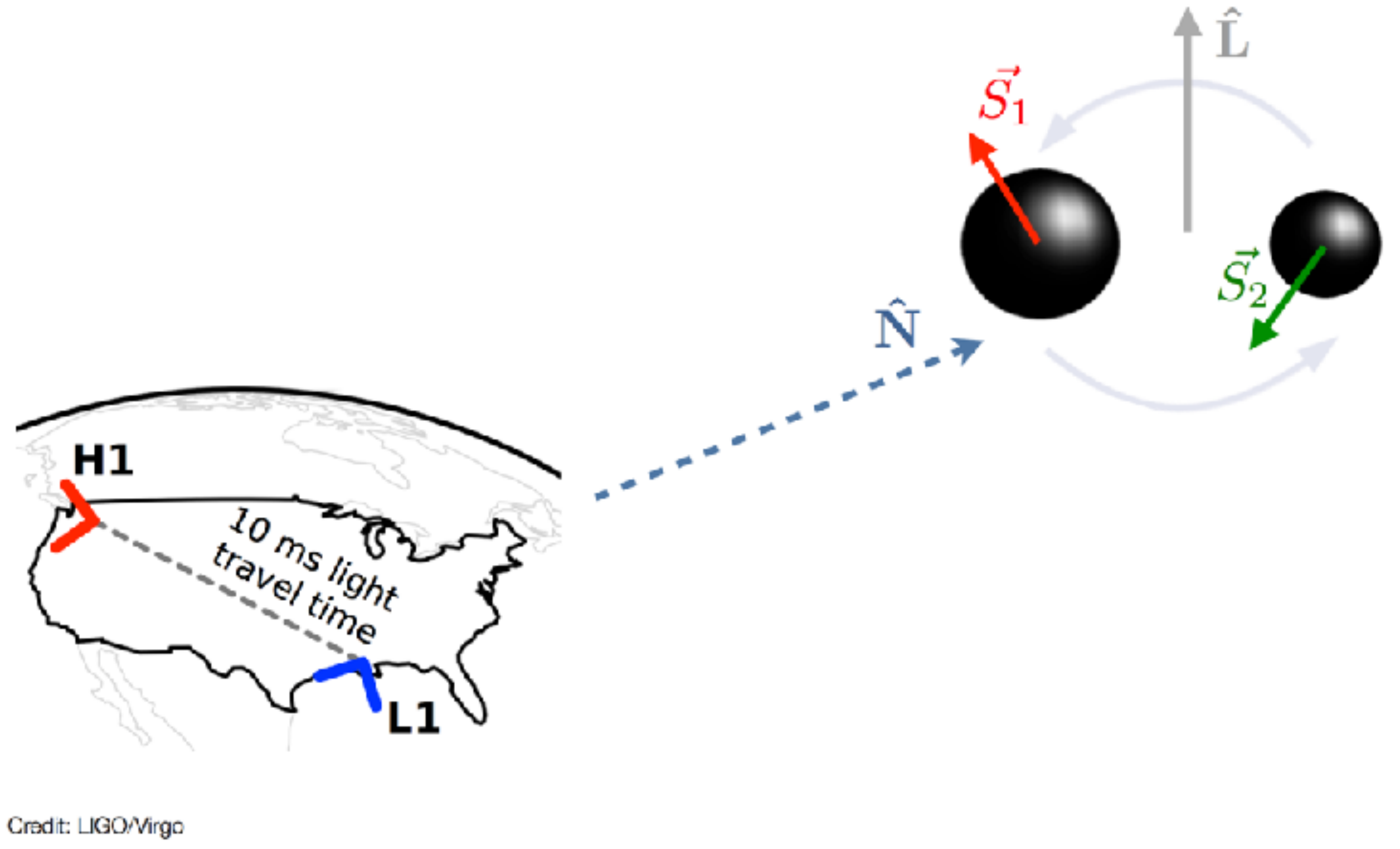
- Need a signal model $h(\theta)$ to compare against the data when evaluating the likelihood

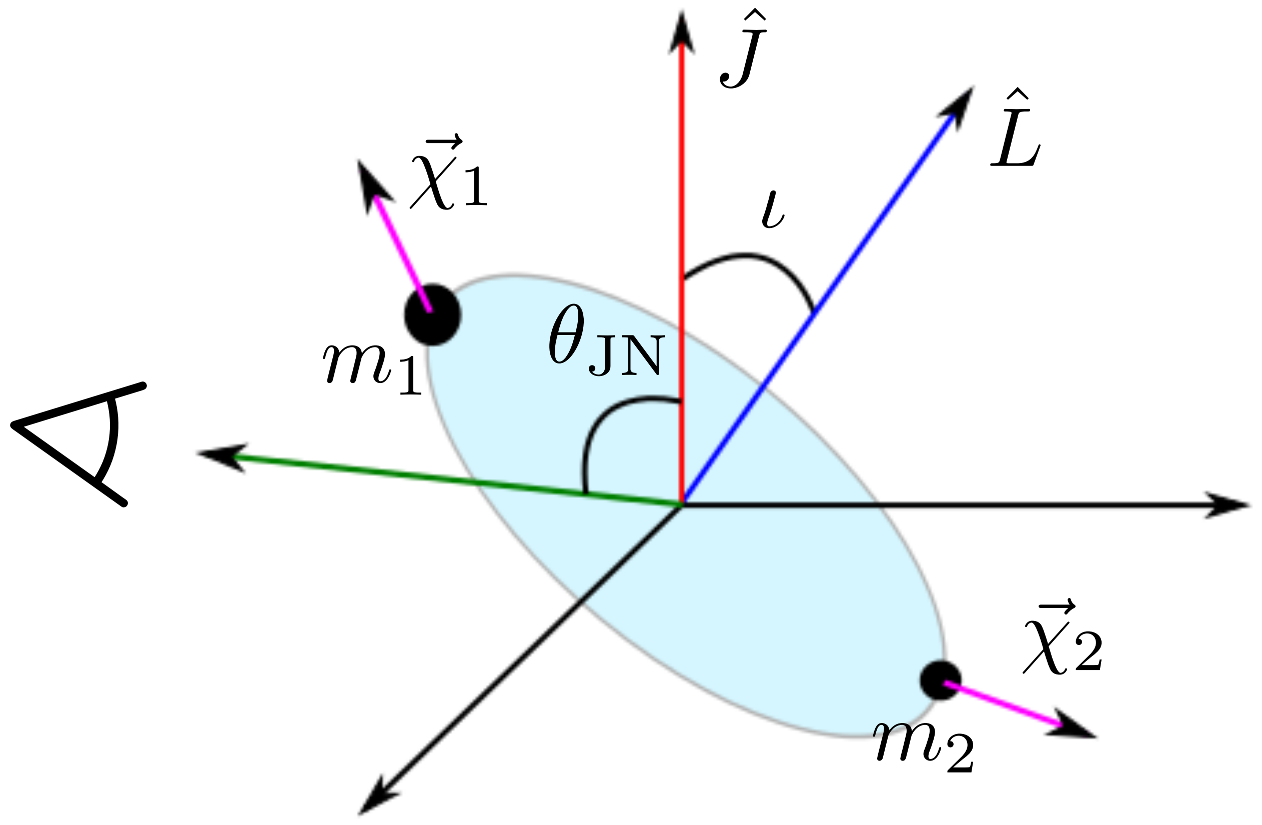
- Circular binary black hole (15D):

$$\theta_{\rm BBH} = \{\underbrace{m_1, m_2, \vec{\chi}_1, \vec{\chi}_2}_{\text{intrinsic}}, \underbrace{D_L, \iota, \alpha, \delta, \psi, \phi_c, t_c}_{\text{extrinsic}}\}$$

- Eccentric orbits: requires two additional parameters to describe the ellipse and its orientation

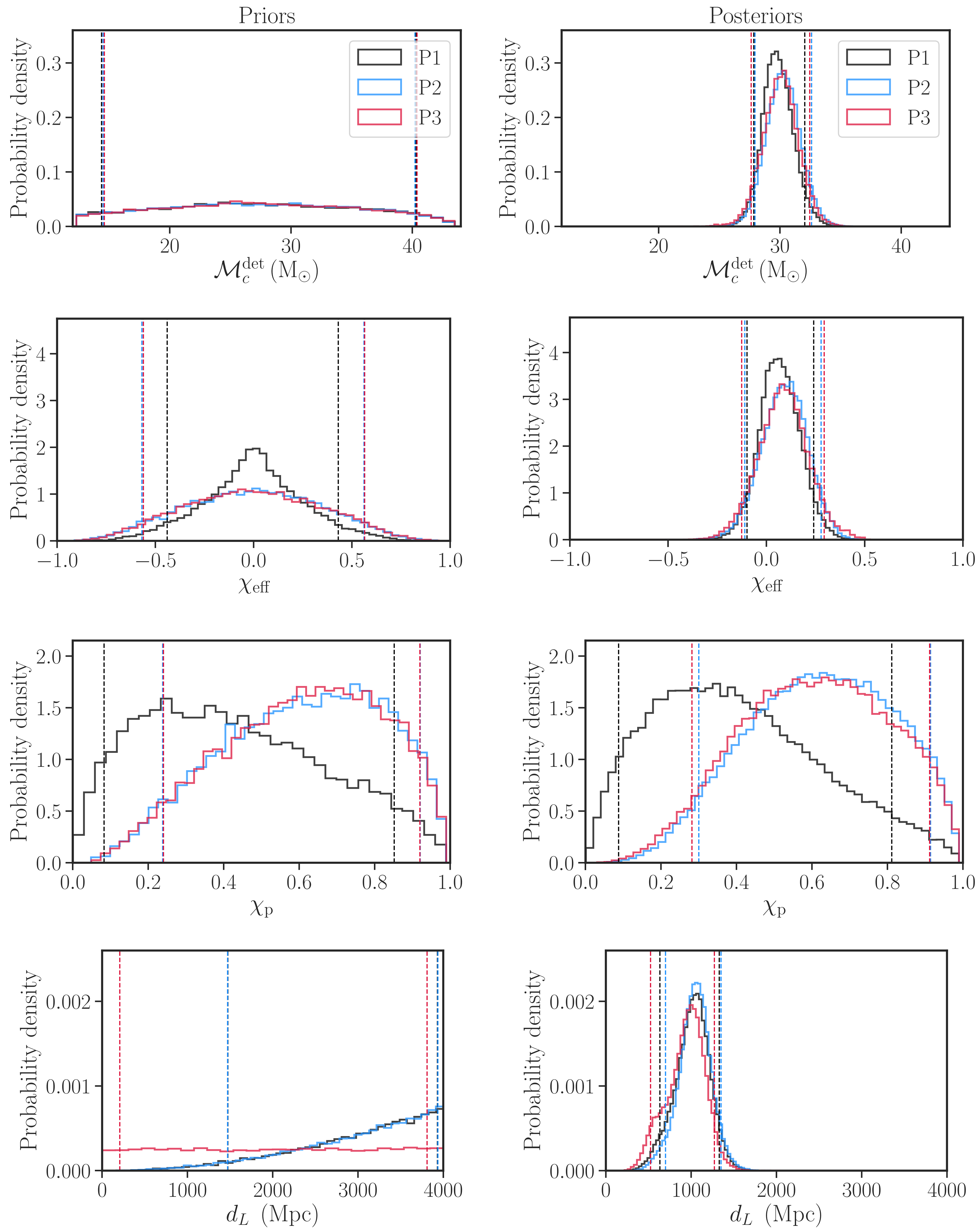
- Binary neutron star: $\theta_{\rm BBH}$ + parameters that characterise the tidal response of the star





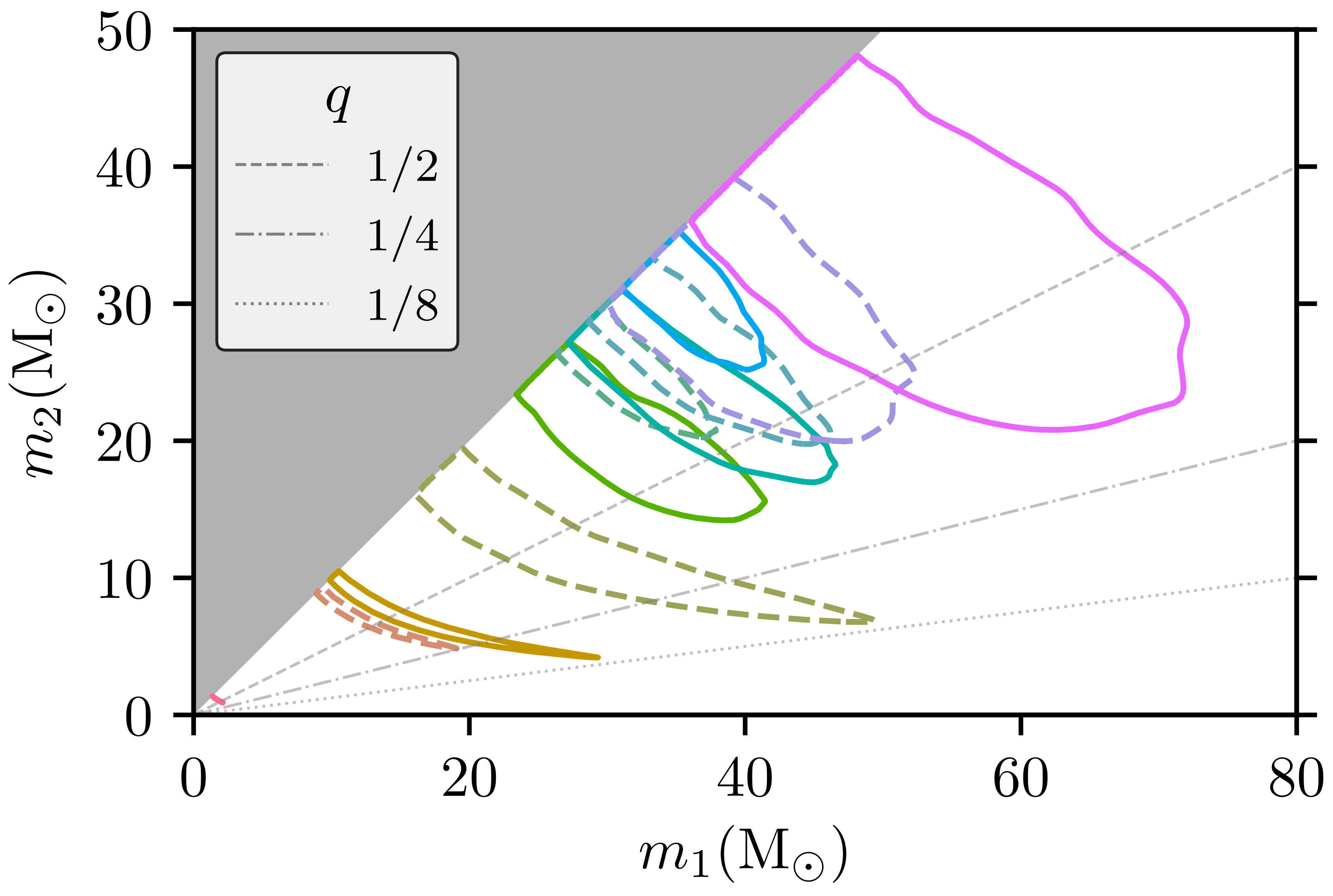
THE PRIOR

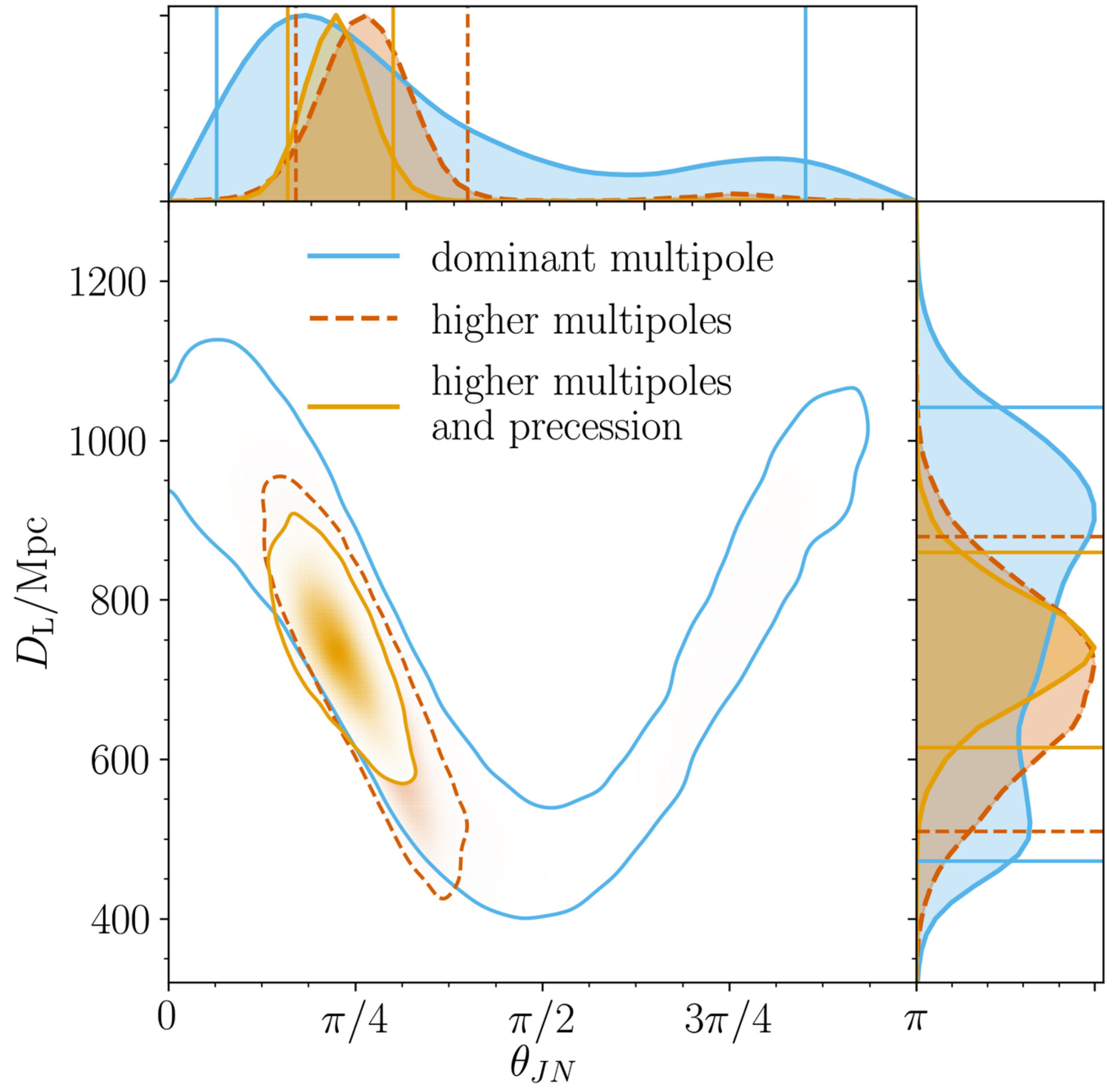
- Uninformative vs. astrophysically motivated priors
  - Population priors
- "Standard" GW priors:
  - Mass priors: often sample uniform in chirp mass $\mathscr{M}_c = \eta^{3/5} M$ & symmetric mass ratio $\eta = (m_1 m_2)/M$
  - Distance: $p(D_L) \propto D_L^2$ (uniform in "luminosity" volume)
  - Spins magnitudes: uniform
  - Spin orientation: isotropic (i.e. uniform in cos(tilt)

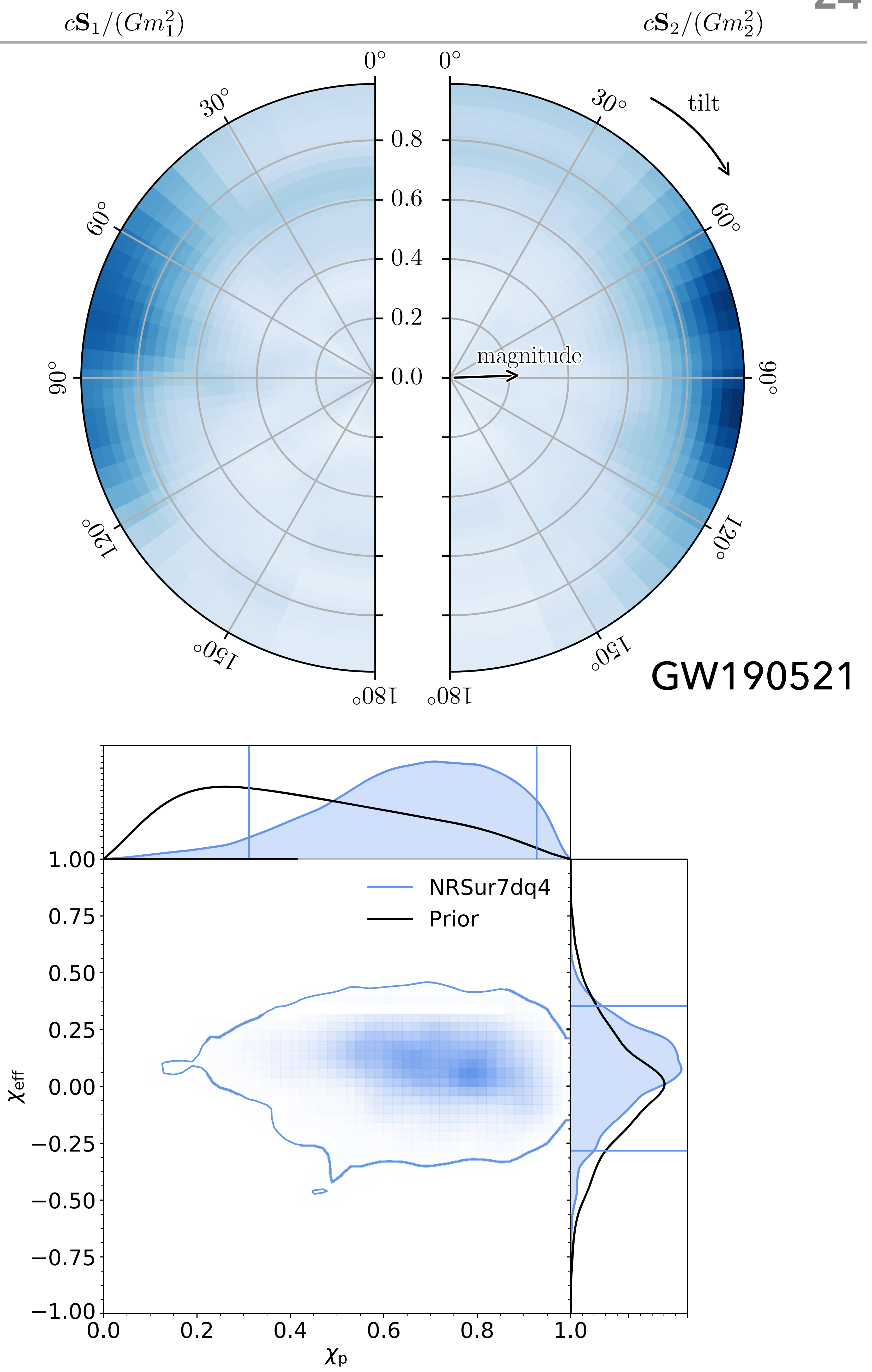


[LVC, GWTC-1]

MEASUREMENT EXAMPLES







- Chirp mass vs total mass
- Distance-inclination degeneracy
- Individual spins vs effective spins
- Impact of SNR (statistical uncertainty)

[LVK: GWTC-1, GW190412, GW190521]

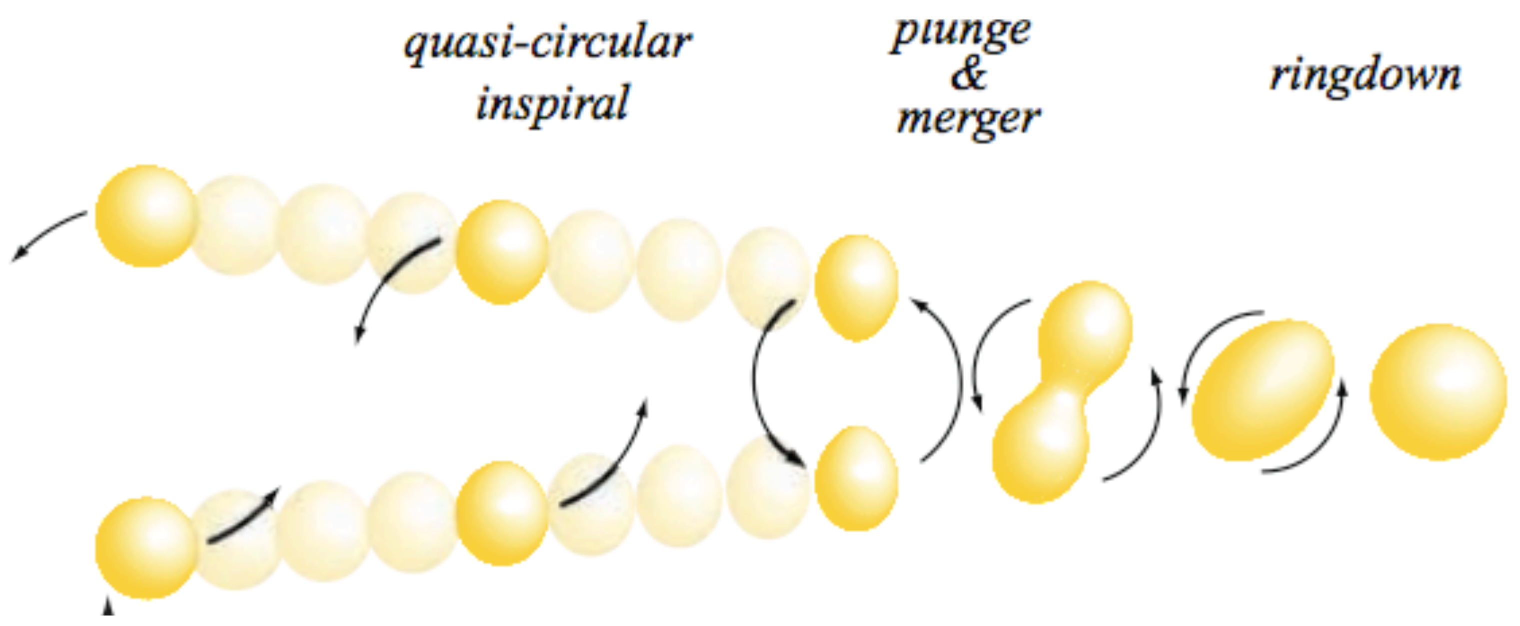
TRY IT YOURSELF :-)

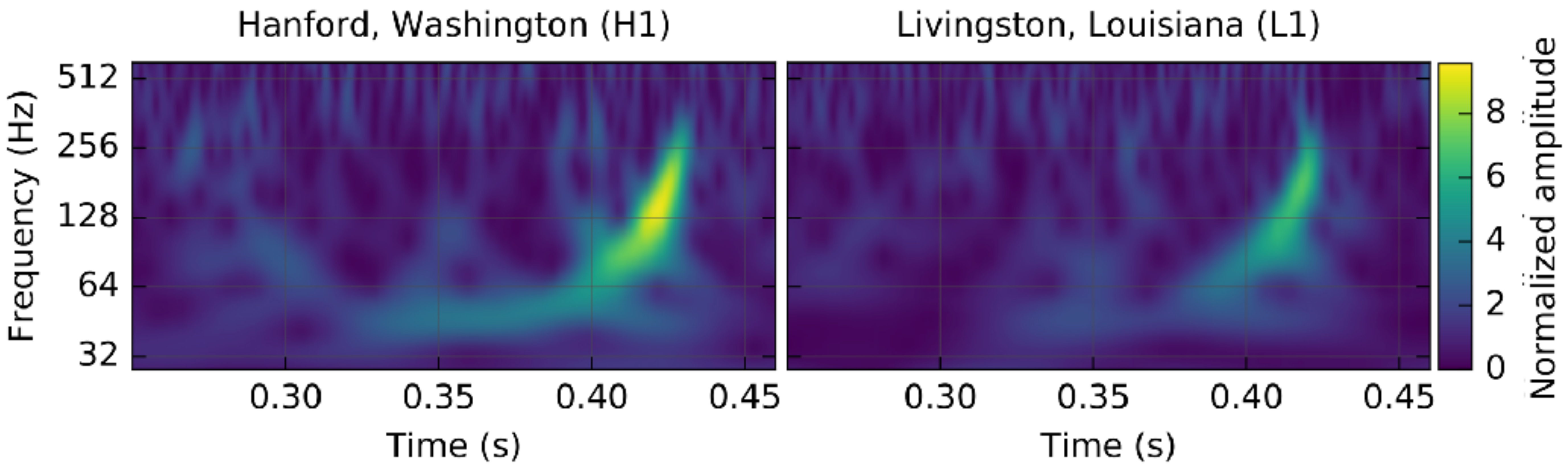
- All LVK analysis software is publicly available: https://git.ligo.org/lscsoft/lalsuite
- Tutorials available from the Gravitational Wave Open Science Centre: https://gwosc.org/tutorials/
- Perform a matched filter search: https://pycbc.org/
- User-friendly Python inference package Bilby:
  - Source code: https://git.ligo.org/lscsoft/bilby
  - Documentation: https://lscsoft.docs.ligo.org/bilby/

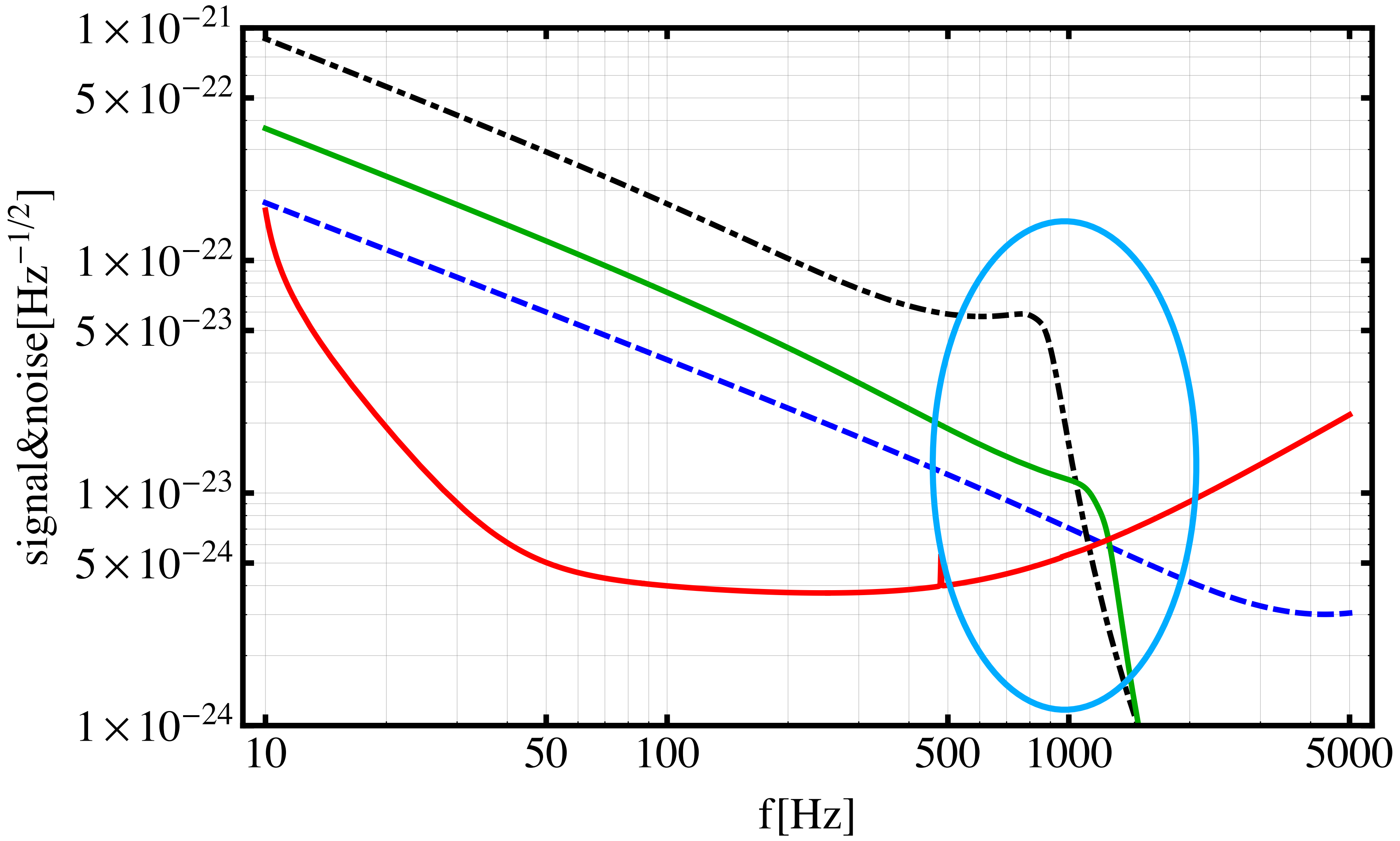
GRAVITATIONAL WAVES FROM COMPACT BINARIES

- Signal „sweeps“ through the detector’s sensitivity band

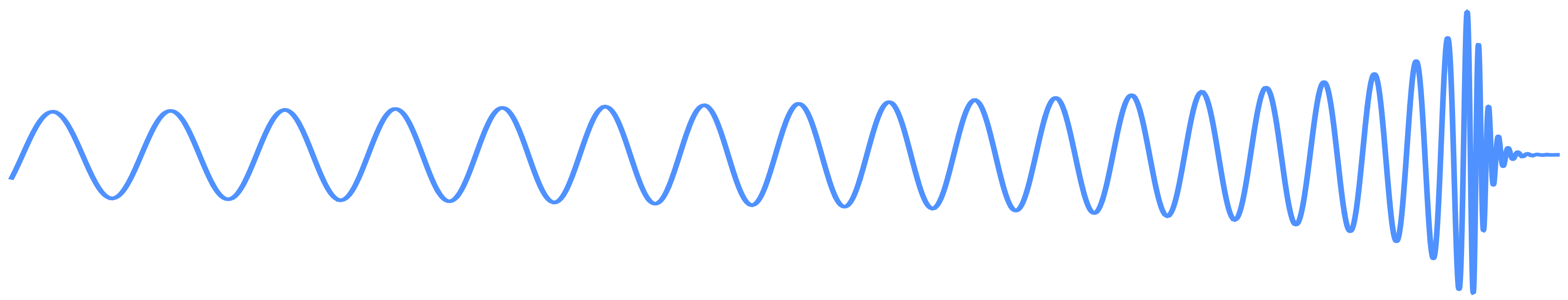
- Depending on the total mass of the binary, the merger regime is visible
    - Inspiral-merger-ringdown (IMR) waveforms are key
    - Need accurate theoretical model to infer the science



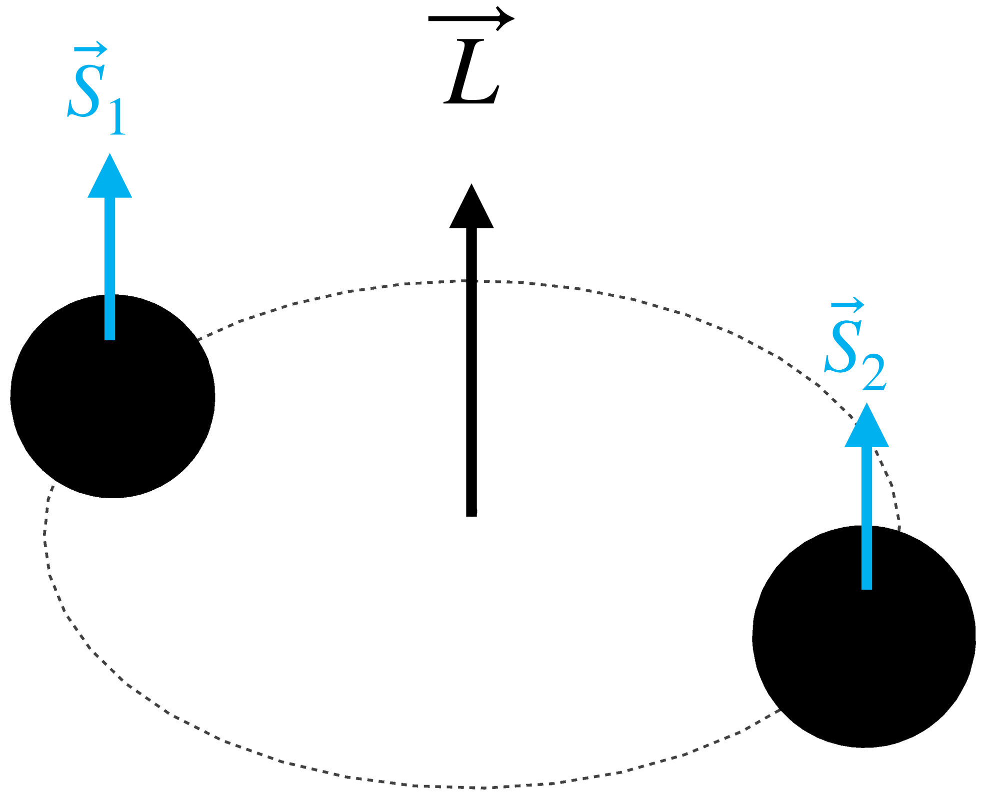




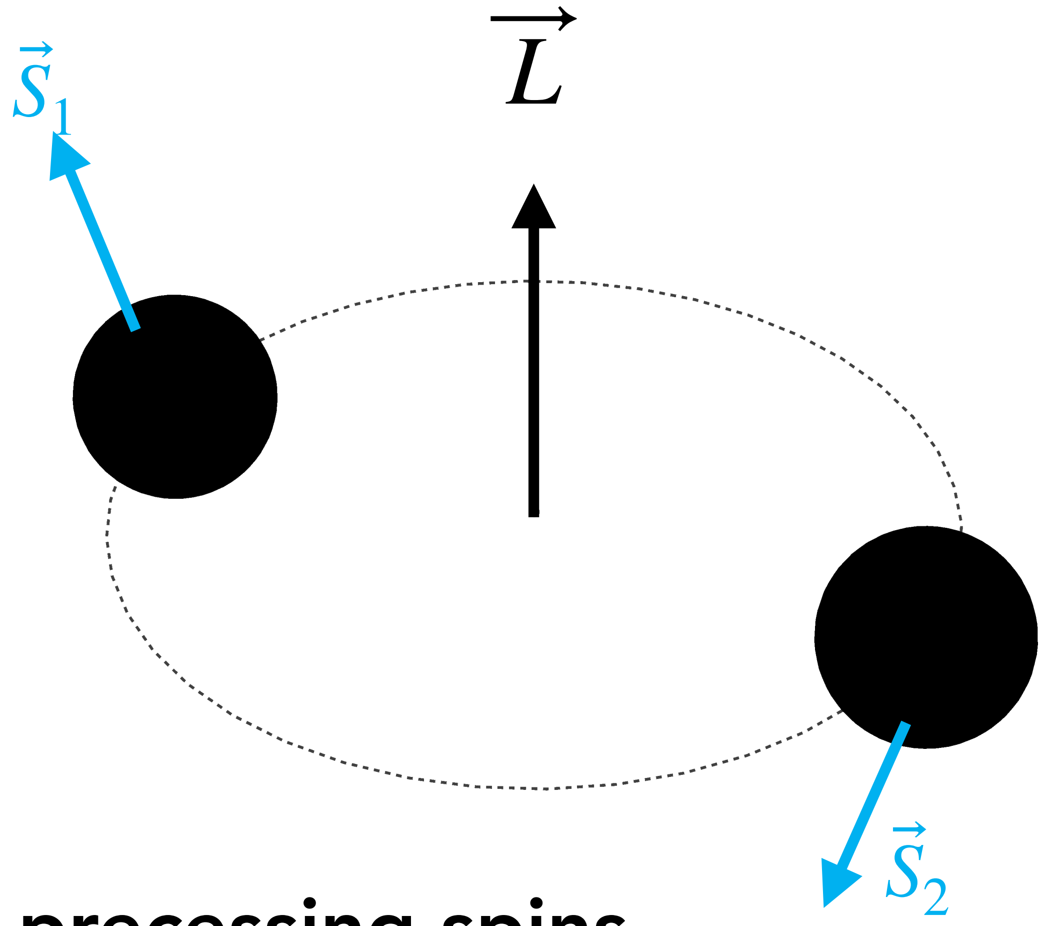
“chirp” - encodes the fundamental source properties

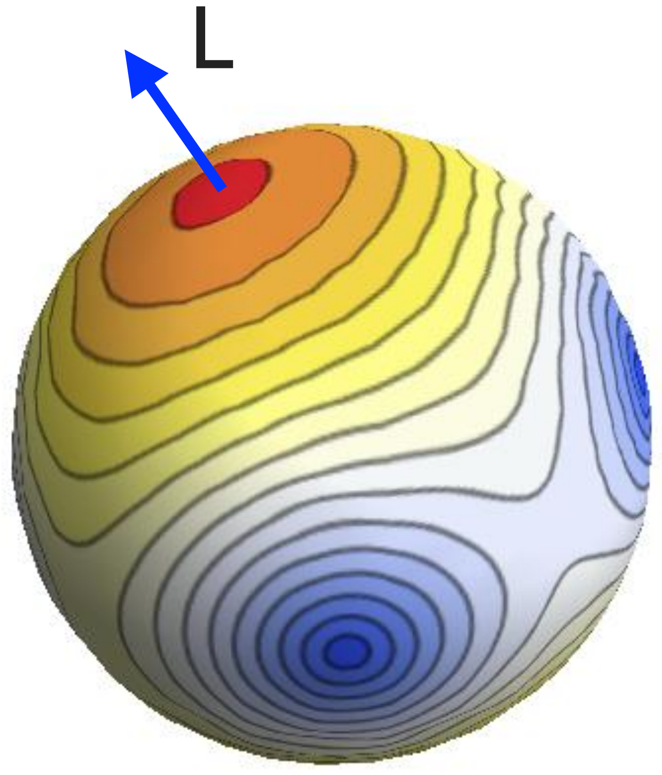


PHENOMENOLOGY

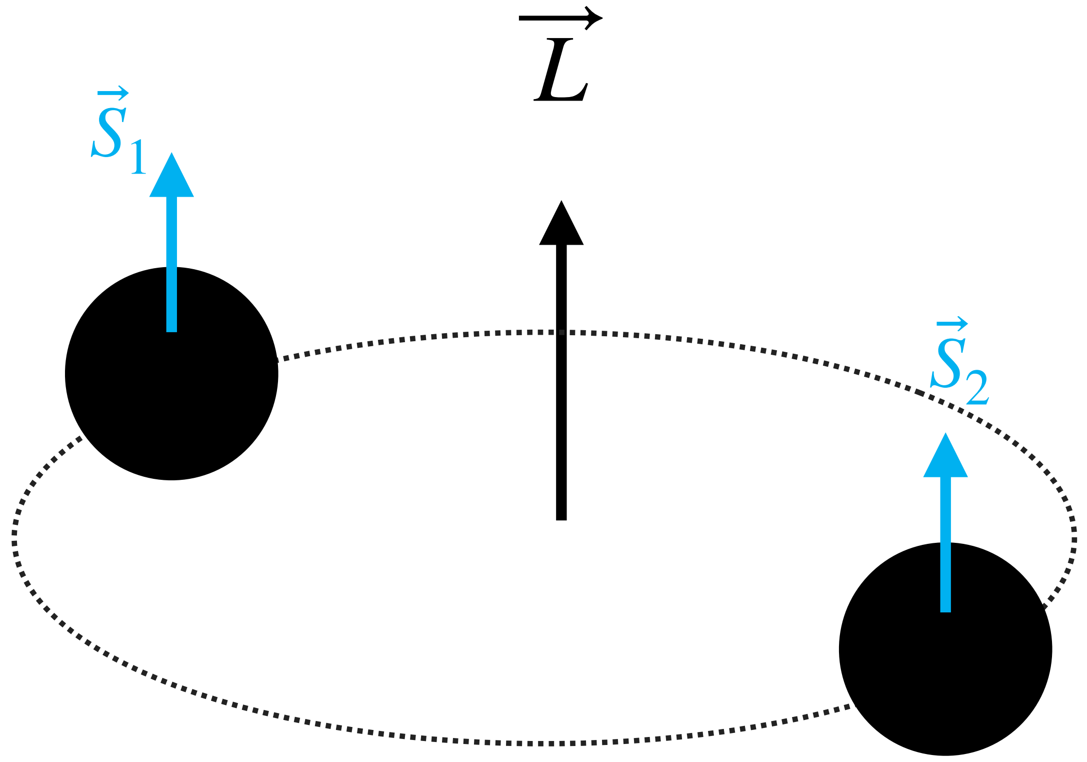


no spins or aligned spins

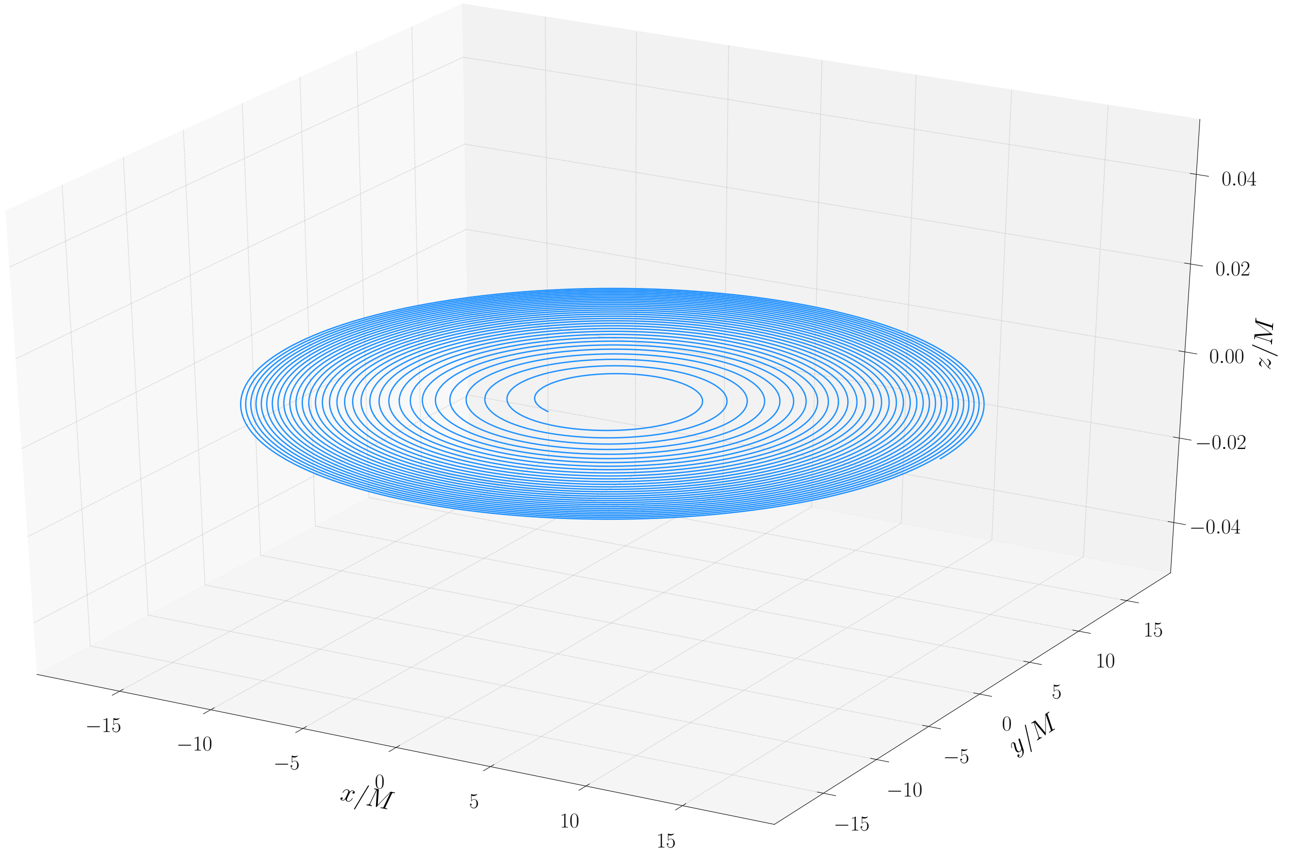


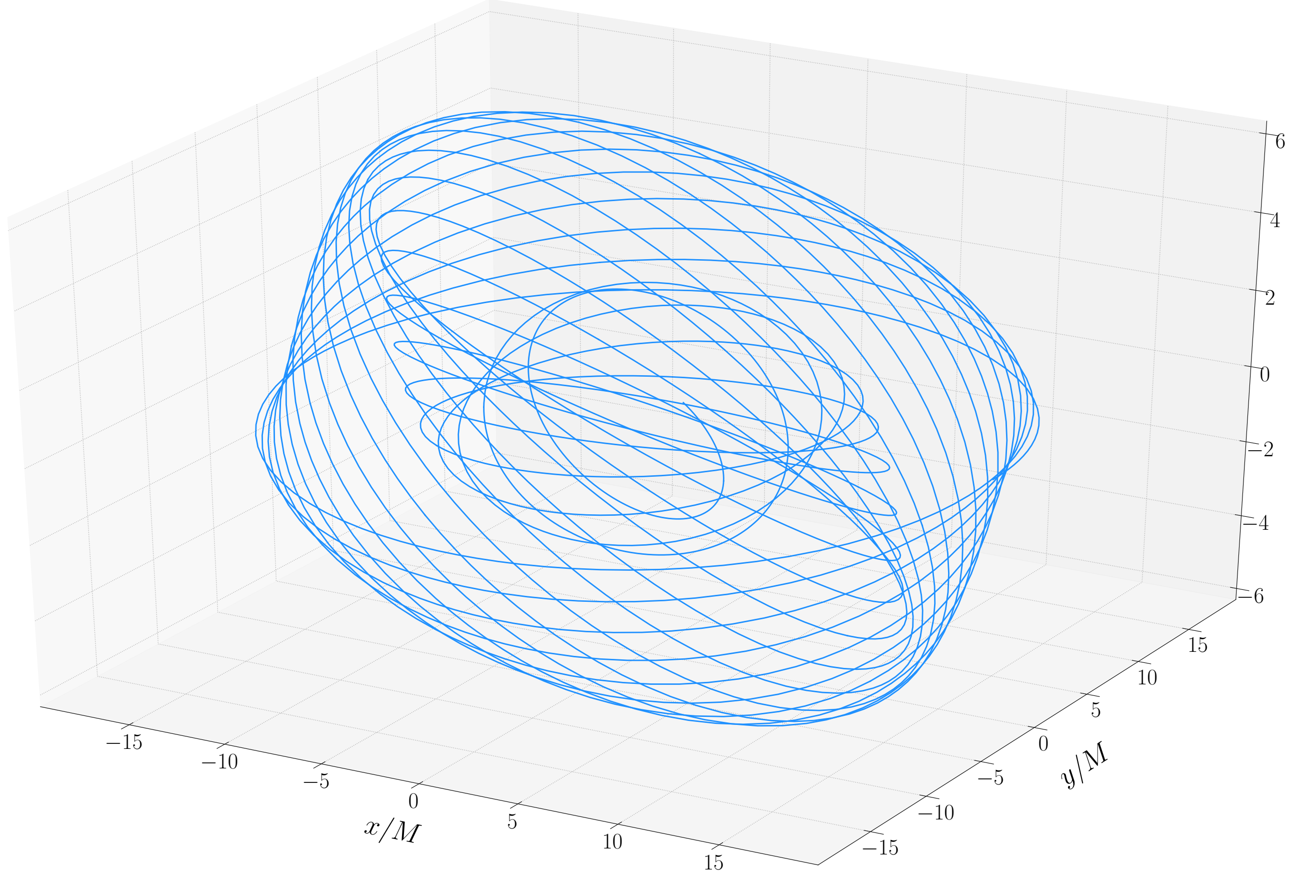


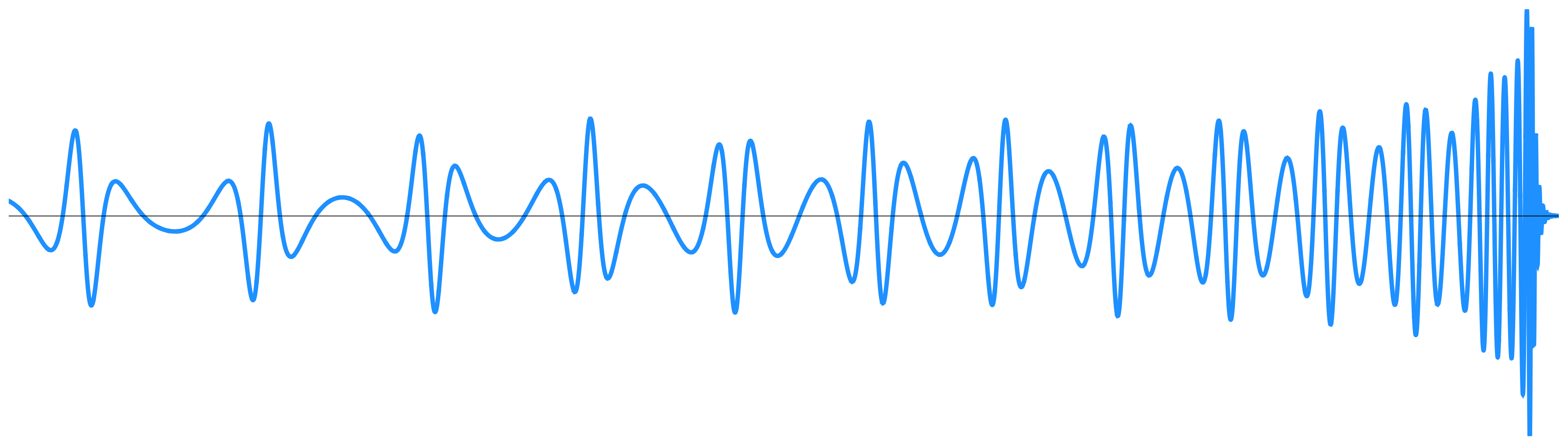
precessing spins

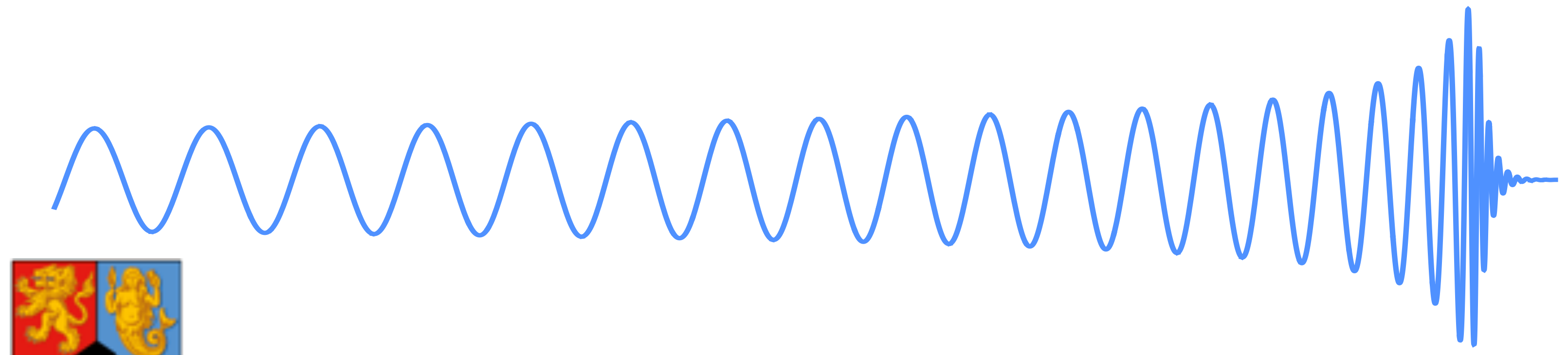


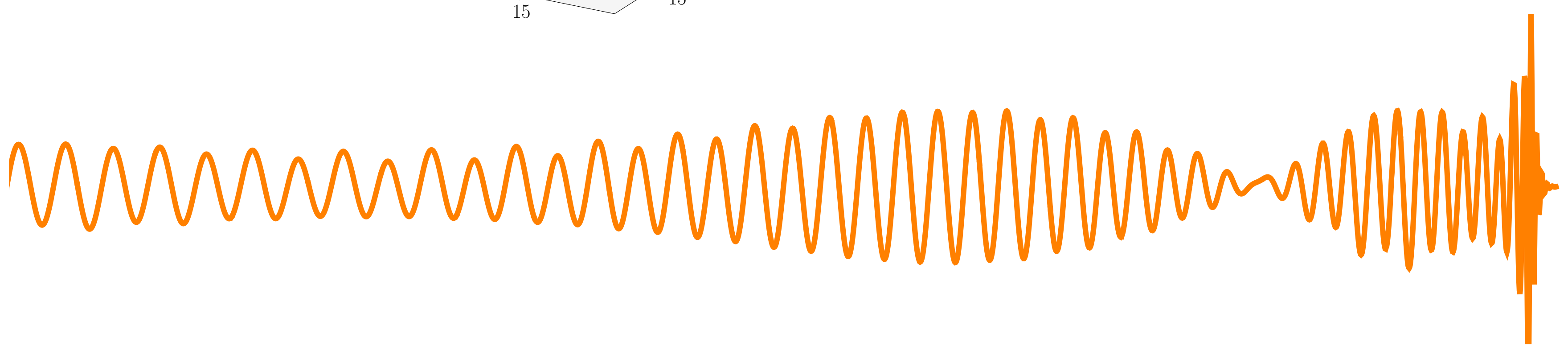
orbital eccentricity



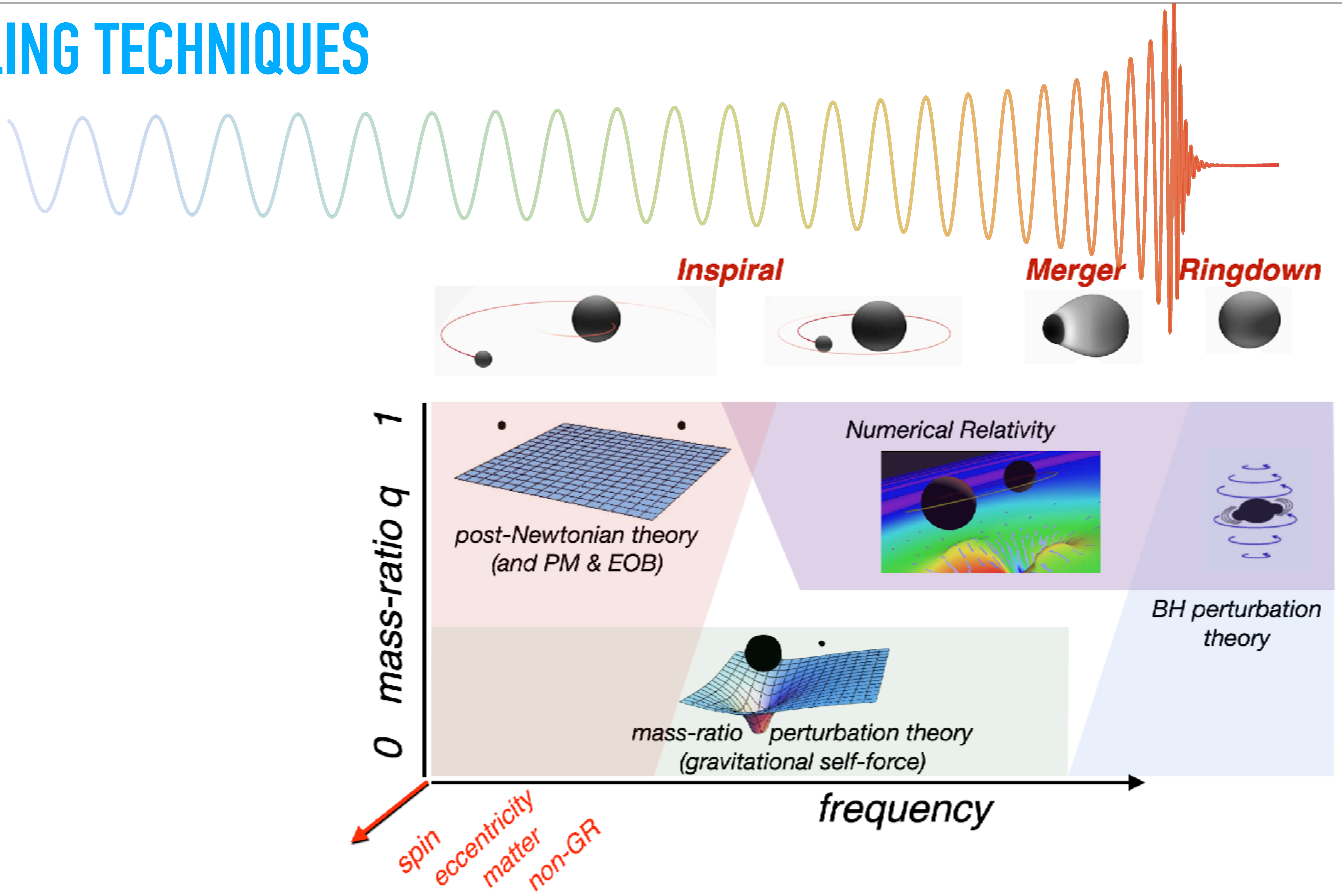








MODELLING TECHNIQUES



Courtesy: H. Pfeiffer

ANALYTICAL RELATIVITY TOOLS

- **Post-Newtonian (PN) theory**
  - Weak fields & slow motion
- **Post-Minkowskian (PM) theory**
  - Weak fields, arbitrary velocities
- (uncalibrated) **effective-one-body** (EOB)
- **Effective field theory (EFT)**
- **Scattering amplitudes**
  - Hyperbolic motion
  - Map to the bound case
- **Gravitational self-force (GSF)**
  - Very unequal mass ratios
- (Classical) **Perturbation theory**
  - Ringdown

"Tutti Frutti"

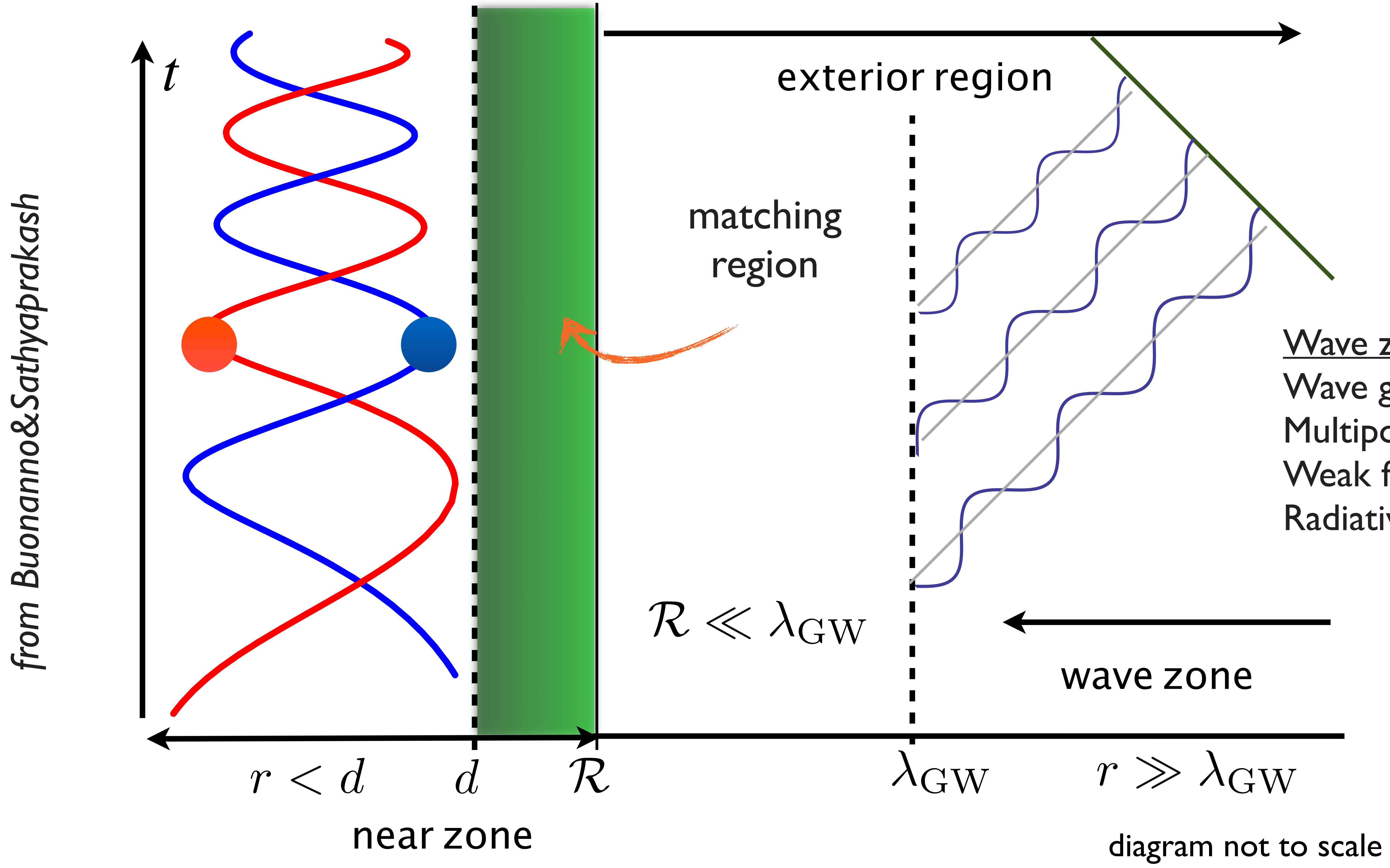
MODELLING THE INSPIRAL

[See Blanchet's LRR review]

- Multi-length-scale problem

<u>Near zone:</u>
Equations of motion
Post-Newtonian
Weak fields & slow motion
Source multipoles

$$\frac{v^2}{c^2} \sim \frac{GM}{r} \ll 1$$



<u>Wave zone:</u>
Wave generation
Multipolar post-Minkowskian in G
Weak fields
Radiative multipoles

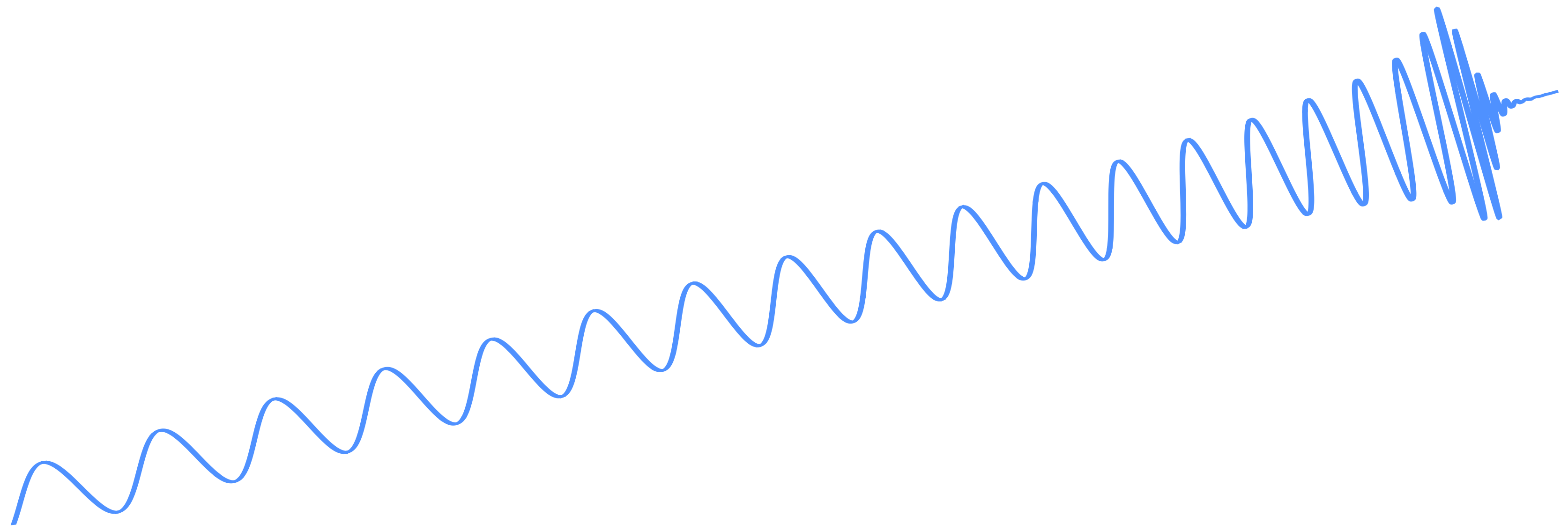
GRAVITATIONAL WAVEFORMS

- A gravitational-wave signal can be written as:

Phase

$$h(f) = \mathcal{A}(f)\, e^{i\varphi(f)}$$

Amplitude



- The GW phase encodes the physics:

0PN 1PN 2.5PN(l) 5PN

$$\varphi(v) = \left(\frac{v}{c}\right)^{-5}\left[\varphi_0 + \varphi_2\left(\frac{v}{c}\right)^2 + \cdots + \varphi_{5l}\ln\left(\frac{v}{c}\right)\left(\frac{v}{c}\right)^5 + \cdots + \varphi_{10}\left(\frac{v}{c}\right)^{10} + \cdots\right]$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} \qquad q = \frac{m_2}{m_1}$$

Spin effects at 1.5PN

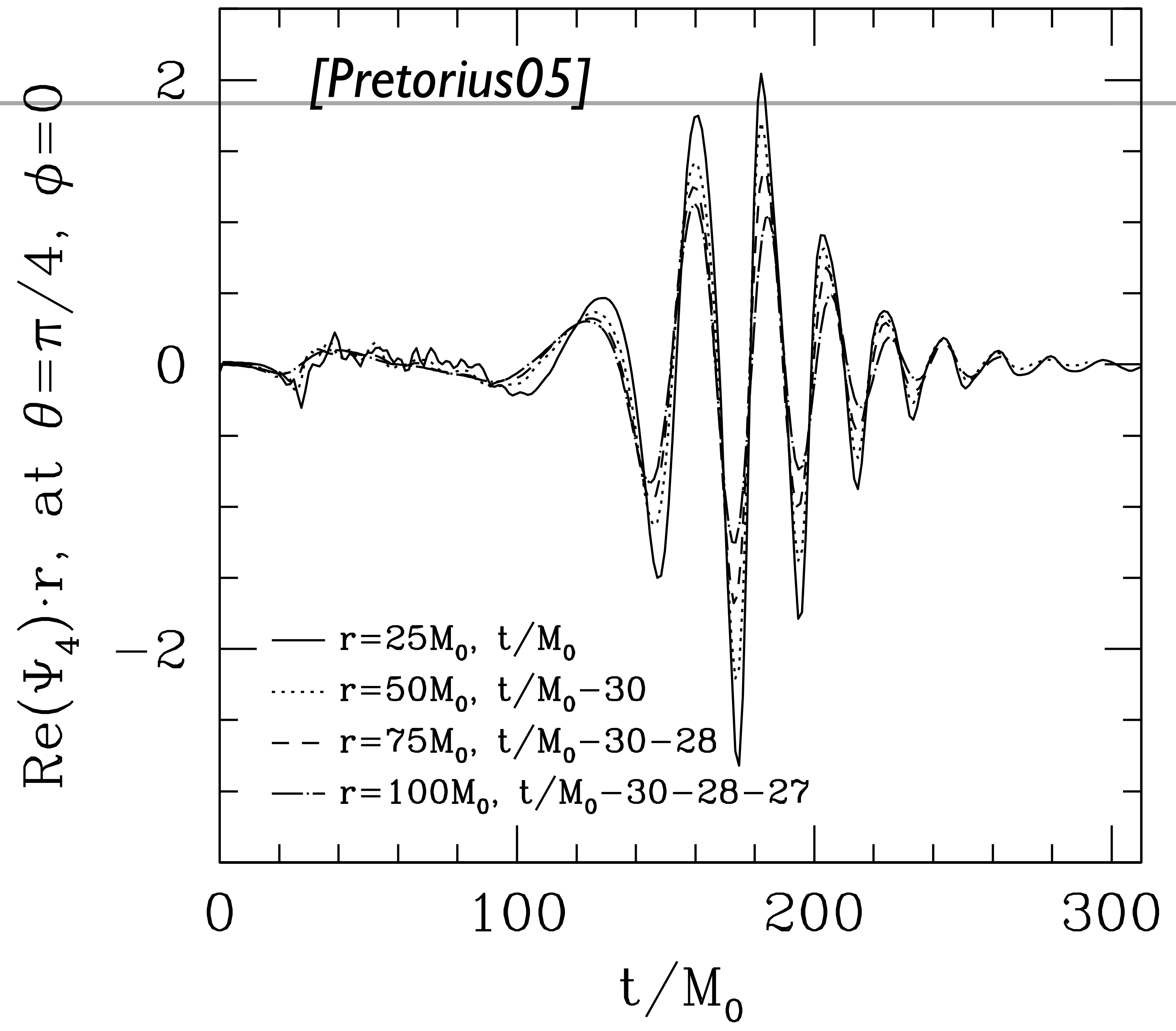
Tail terms ~ back reaction due to scattering

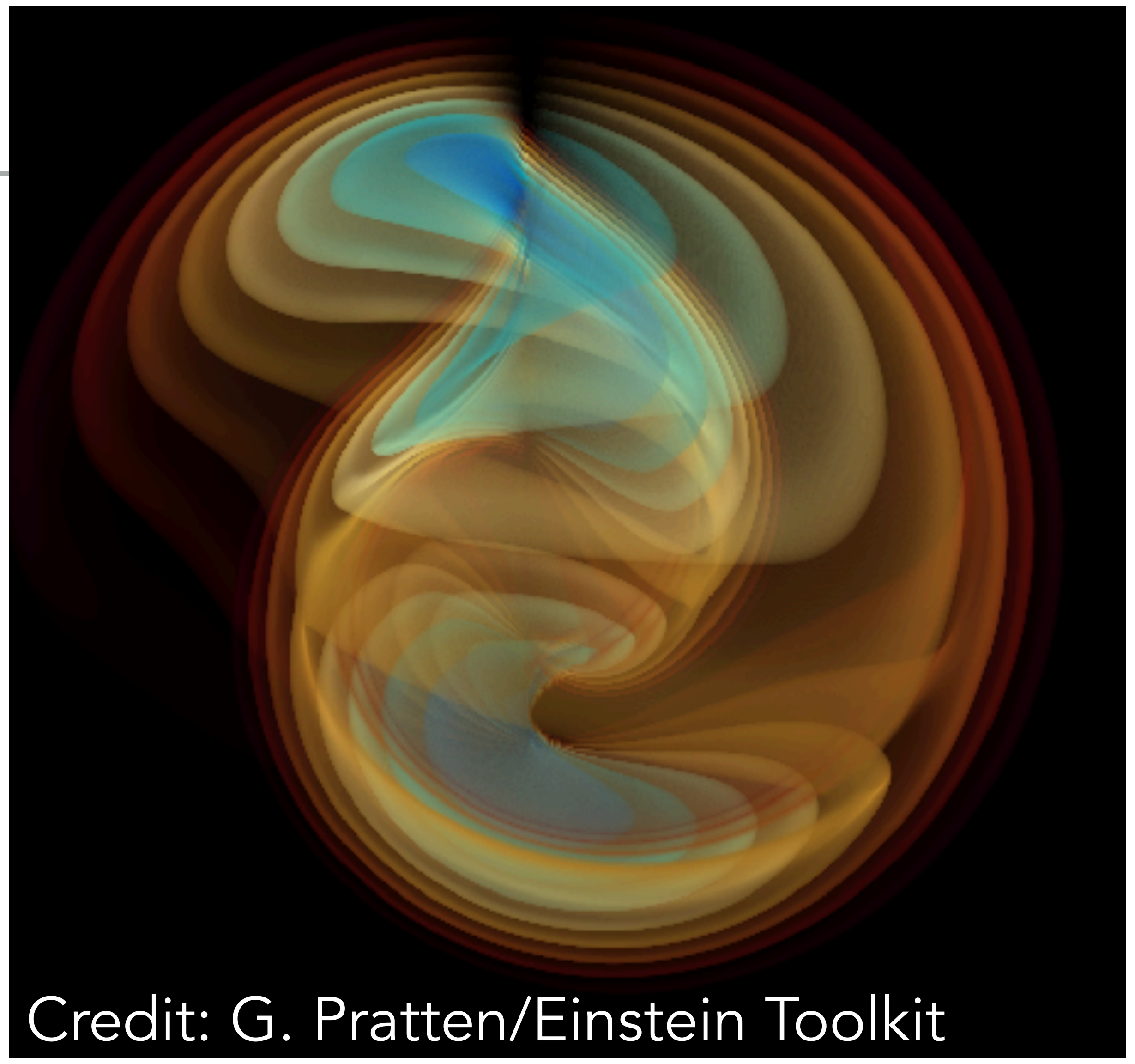
Tidal effects first enter here…

Slide credit: G. Pratten

NUMERICAL RELATIVITY (NR)

- Only few analytic solutions to Einstein field equations known
  - **No analytic solutions to the general-relativistic two-body problem**
  - NR is a key ingredient to understanding GW observations!
- Breakthrough in 2005 *[Pretorius, Baker+, Campanelli+]*:
  - First stable binary evolutions
  - Extraction of the GW signal
  - Computationally expensive
  - Time consuming
- Many challenges remain!





Reformulate the Einstein field equations (EFE) as initial value problem (IVP)

Prove existence of a well-posed initial value problem

Numerically suitable reformulation of the EFE

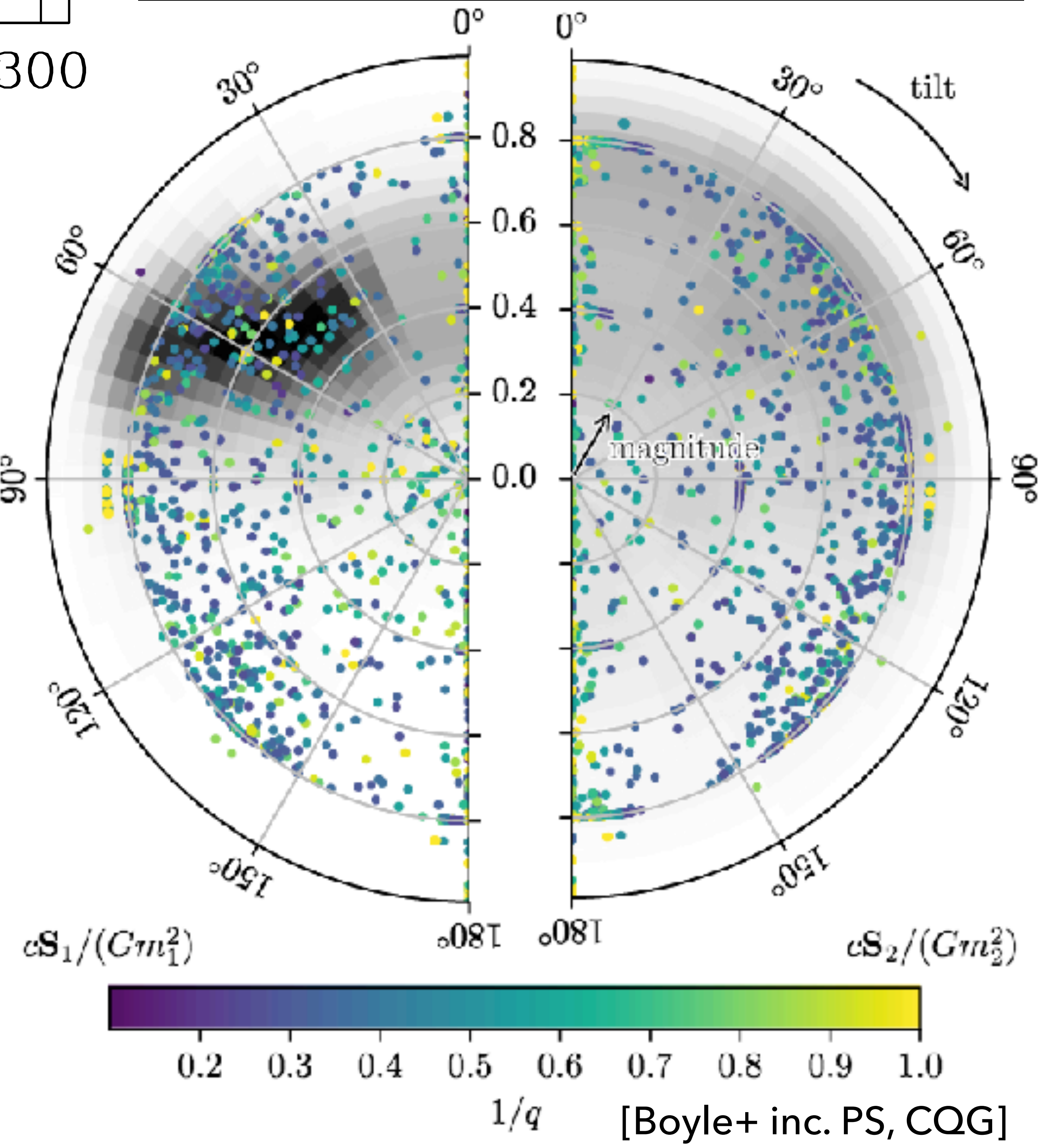
"good" coordinates (gauge choices)

Initial data

Deal with singularities

"Find" the black hole horizons

Extract gravitational waves



[Boyle+ inc. PS, CQG]