Lectures on Cosmic-Ray Theory

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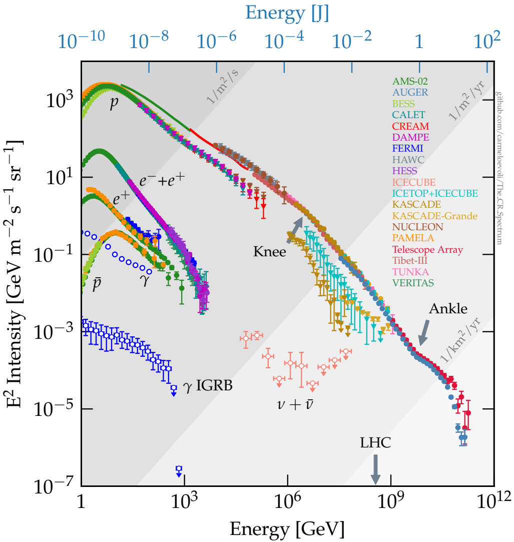
The cosmic-ray spectrum in 2024

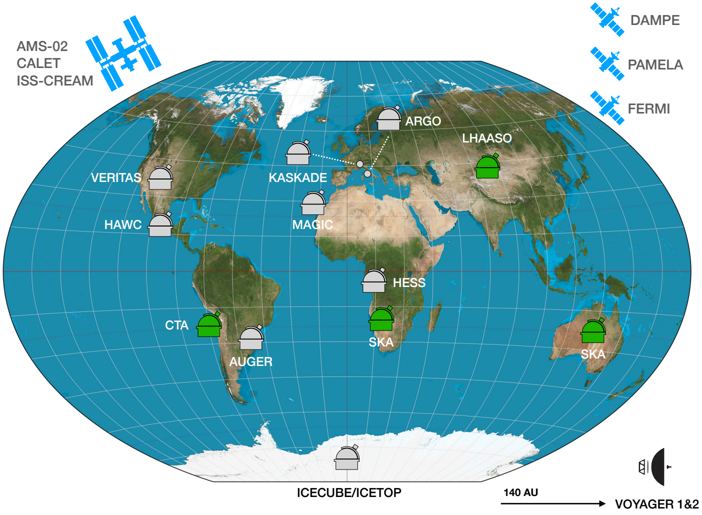


"There are two big power laws in the sky..." (J.R. Jokipii)

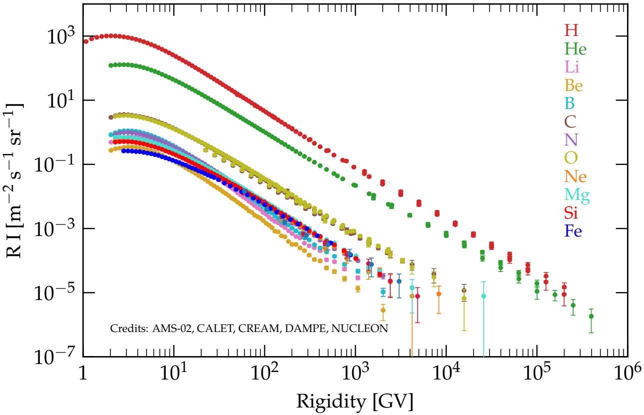
The Non-Thermal Universe

- The non-thermal activity of the Universe can be distilled into two fundamental big Q's:
- How does nature accelerate particles, typically from the thermal pool?
- How do these non-thermal particles propagate through the complex environments inside their sources or from the sources to us?
- Every part of the Universe shows evidence of non-thermal activity, suggesting that the formation of plasma is inherently associated with high-energy particles.
- Understanding these processes and their interconnections is crucial for further investigation.

The non thermal Universe



Galactic Cosmic Rays: unprecedented measurements



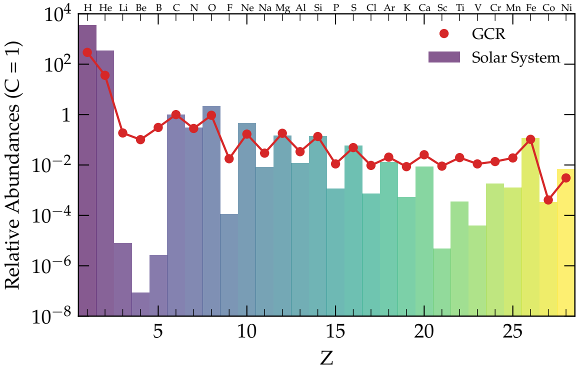
- Amazing new data: The spectrum of each isotope includes contributions from many different parents (both in terms of fragmentation and decays) giving to each observed isotope a potentially very complex history
- In these lectures we will mainly focus on our Galaxy as we take advantage of far more information than any other environment, bearing in mind that the very same physical picture can be straightforwardly applied to the vast majority of astrophysical territories.

Galactic Cosmic Rays: unprecedented measurements

- Amazing new data: The spectrum of each isotope includes contributions from many different parents (both in terms of fragmentation and decays) giving to each observed isotope a potentially very complex history
- In these lectures we will mainly focus on our Galaxy as we take advantage of far more information than any other environment, bearing in mind that the very same physical picture can be straightforwardly applied to the vast majority of astrophysical territories.
- At the end of these lectures you should be able to extract from this plot key quantities on the properties of CR sources, ISM plasmas, and so on...

More lecture notes and exercises here: https://github.com/carmeloevoli/heath-2324 (pay attention of several ~~mistakes~~ typos!)

Basic indicators of diffusive transport: stable elements



- Thermal particles in the average interstellar medium are somehow accelerated to relativistic energies becoming CRs $\longrightarrow$ primary CRs
- It must exist also a second population which is produced during propagation by primary fragmentation $\longrightarrow$ secondary CRs

Basic definitions: The grammage pillar

- The grammage X is the amount of material that the particle go trough along propagation (a sort of column density):

$$X = \int dl \rho(l)$$

- I assume a simple system with one primary species n_p and one secondary n_s only.
- The evolution of primary and secondary along the grammage trajectory is given respectively by:

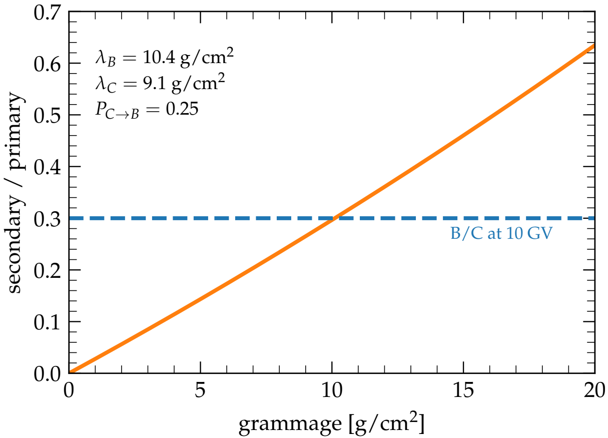
$$\begin{aligned} \frac{dn_p}{dX} &= -\frac{n_p}{\lambda_p} \\ \frac{dn_s}{dX} &= -\frac{n_s}{\lambda_s} + P_{p\to s}\frac{n_p}{\lambda_p} \end{aligned}$$

  with initial conditions $n_p(X=0)=n_0$ and $n_s(X=0)=0$, where λ_i are some kind of interaction lenght (probability) and P is the fraction resulting in that specific channel.
- Solving this, I can get n_s/n_p in terms of X, λ_s and λ_p only:

$$\frac{n_s}{n_p} = P_{p\to s}\frac{\lambda_s}{\lambda_s-\lambda_p}\left[\exp\left(-\frac{X}{\lambda_s}+\frac{X}{\lambda_p}\right)-1\right]$$

$\to$ I quantify the transport process, whatever it is, in something that can be either directly measured in CRs n_s/n_p or provided by a nuclear physics experiment (λ's, P's).

Basic definitions: The grammage pillar



$$\text{B/C} \sim 0.3 \longrightarrow X = 10\,\text{g/cm}^2$$

Basic definitions: The grammage pillar

- Let me assume that the grammage is accumulated in the gas disc of our Galaxy
- At each crossing of the disc $n_{\rm gas} \sim 1$ cm^{-3}, $h \sim 200$ pc:

$$\chi_d \sim m_p n_{\rm gas} h_d \sim 10^{-3}\,{\rm g/cm^2} \ll \chi_{\rm B/C}$$

- The grammage accumulated in one crossing is clearly inconsistent with the grammage we estimate from CR measurements $\longrightarrow$ the particles have to cross the disk many times
- The time spent in the gas region before escaping the Galaxy must be not less than:

$$t_{\rm esc,min} \sim \frac{\chi_{\rm B/C}}{\chi_d}\frac{h}{v} \sim 7 \times 10^6\,{\rm years} \gg \frac{\rm kpc}{c}$$

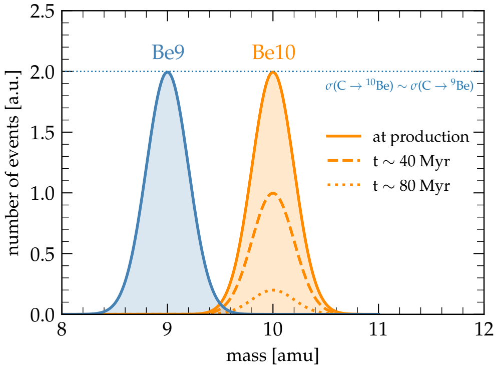
which exceeds by order of magnitudes any possible ballistic timescale in the MW $\sim \mathcal{O}(\frac{\rm kpc}{c})$

- We deduce that CRs follow something more similar to a Brownian motion in the Galaxy

Key question!

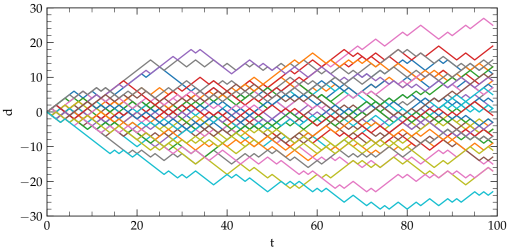
What is the origin of confinement of these particles in the Galaxy?

Basic indicators of diffusive transport: unstable elements



- ^{10}Be is a β^- unstable isotope decaying in ^{10}B with an half-life of ~ 1.5 Myr
- Similar production rates than other (stable) isotopes $\sigma_{\rm Be9} \sim \sigma_{\rm Be10}$
- Traditionally ^{9}Be/^{10}Be has been used as CR clock pointing to a residence time of $\mathcal{O}(100)$ Myr

Basic definitions: random walk and diffusion coefficient



- After N steps $\vec{\lambda}_i$ of the same size $||\lambda_i|| = \lambda$ and random direction a particle has reached a distance:

$$\vec{d} = \sum_{i=1}^{N} \vec{\lambda}_i$$

- The scalar product of d with itself is

$$\vec{d} \cdot \vec{d} = \sum_{i=1}^{N} \sum_{j=1}^{N} \vec{\lambda}_i \cdot \vec{\lambda}_j \longrightarrow d^2 = N\lambda^2 + 2\lambda^2 \sum_{i=1}^{N} \sum_{j<1}^{N} \cos\theta_{ij} \sim N\lambda^2 \rightarrow \langle d \rangle \simeq \sqrt{N}\lambda$$

as we assumed that the angles θ_{ij} are chosen randomly and thus the off-diagonal terms are uncorrelated.

Basic definitions: random walk and diffusion coefficient

- The continuity equation for the number density n and its current $\vec{j}$ reads

$$\frac{\partial n}{\partial t} + \nabla \vec{j} = q$$

assuming q to be the sum of all sources or losses.

- Combined together with Fick's law for an isotropic flux $\vec{j} = -D\nabla n$ leads to the diffusion equation:

$$\frac{\partial n}{\partial t} - \nabla(D\nabla n) = q$$

- The propagator (Green function) of the 1D diffusion equation with constant D is

$$G(d) = \frac{1}{(4\pi Dt)^{1/2}} \mathrm{e}^{-\frac{d^2}{4Dt}}$$

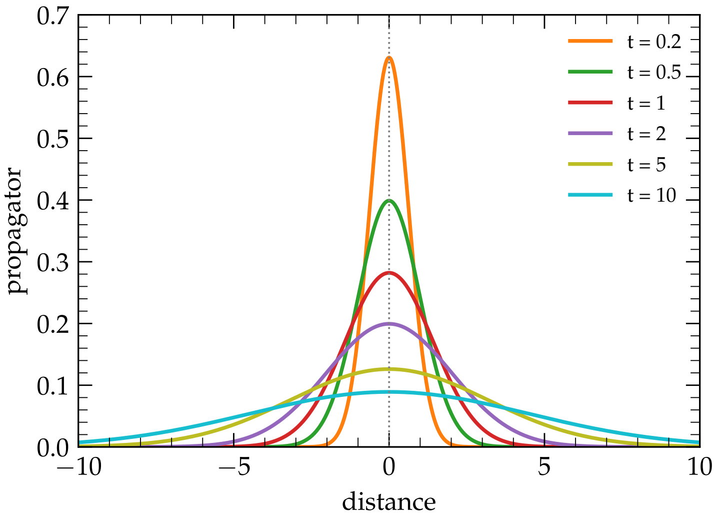
thus the mean distance traveled outward is $\langle d \rangle \simeq \sqrt{Dt}$

- Connecting the two pictures we obtain that D is the product of particle velocity v and mean free path λ:

$$D \sim \frac{N\lambda^2}{t} \sim \frac{v\lambda}{3}$$

where the numerical factor is obtained in 3D with a more accurate derivation.

Basic definitions: random walk and diffusion coefficient



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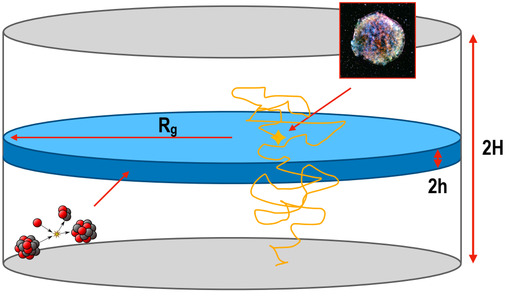
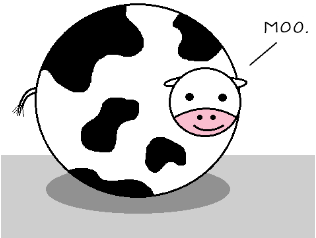
A toy model for protons in our Galaxy: main assumptions



- In the standard model for the origin of Galactic CRs, these particles are accelerated in the disc $h \sim 100$ pc with an injected spectrum $q_p \propto E^{-\gamma}$ where $\gamma \gtrsim 2$
- after injection, CRs propagate diffusively throughout the Galactic halo $H \sim \mathcal{O}(\text{kpc})$ with a diffusion coefficient $D \propto E^{\delta}$ where $\delta \sim 1/3 - 1/2$ and free escape at the boundaries.
- $R_{\rm g} \gg H$ is the radius of the Galactic disc $\rightarrow$ 1D problem
- Secondary production, e.g. LiBeB, takes place predominantly in the disc h where all the gas is confined.

A toy model for protons in our Galaxy



$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial z}\left(j_{\rm diff} + j_{\rm adv} + \dots\right) = \boxed{\text{sources}} - \boxed{\text{losses}}$$

e.g., SNRs, fragmentation, decay, ionization...

A toy model for protons in our Galaxy

- The simplest transport equation for protons, assuming relativistic particles $p \simeq E$:

$$-\frac{\partial}{\partial z}\left[D(E)\frac{\partial n_p}{\partial z}\right] = Q(E,z) = \frac{\xi E_{\rm SN}\mathcal{R}_{\rm SN}}{\pi R_d^2} q_0(E)\delta(z)$$

  where $n_p(E)$ is the cosmic ray density, $E_{\rm SN} \simeq 10^{51}$ erg is the SN kinetic energy converted to proton with efficiency ξ, and $\mathcal{R}_{\rm SN} \simeq 1/100\ {\rm yr}^{-1}$ is the SN galactic rate.

- For $z \neq 0$, and using the boundary condition $n_p(z = \pm H, E) = 0$:

$$j_{\rm diff} = D\frac{\partial n_p}{\partial z} = \text{Constant} \longrightarrow n_p(z) = n_0\left(1 - \frac{z}{H}\right)$$

- Since the diffusive flux is constant in z, in particular at the disc $z = 0$:

$$\left. D\frac{\partial n_p}{\partial z}\right|_{z=0^+} = -D\frac{n_{p,0}}{H}$$

- We now integrate the diffusion equation around $z = 0$

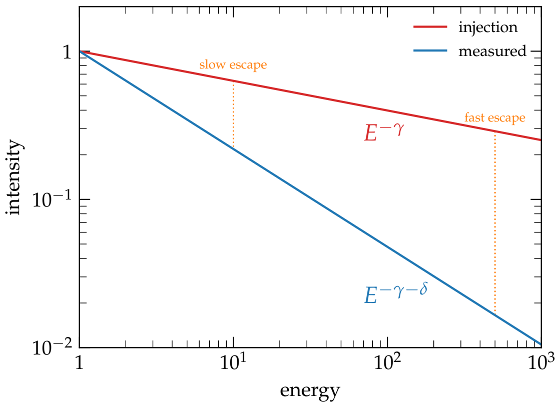
$$\lim_{\epsilon\to 0}\int_{\epsilon^-}^{\epsilon^+} dz \left\{-\frac{\partial}{\partial z}\left[D\frac{\partial n_p}{\partial z}\right] = Q(E,z)\right\} \longrightarrow -2D\left.\frac{\partial n_p}{\partial z}\right|_{z=0^+} = \frac{\xi E_{\rm SN}\mathcal{R}_{\rm SN}}{\pi R_d^2} q_0(E)$$

- and using the equation for the flux:

injection rate per unit volume — escape rate

$$n_p(E) = \frac{E_{\rm SN}\mathcal{R}_{\rm SN} q_0(E)}{2\pi R_d^2}\frac{H}{D(E)} = \boxed{\frac{\xi E_{\rm SN}\mathcal{R}_{\rm SN} q_0(E)}{2\pi R_d^2 H}}\boxed{\frac{H^2}{D(E)}} \propto E^{-\gamma-\delta}$$

A toy model for protons in our Galaxy

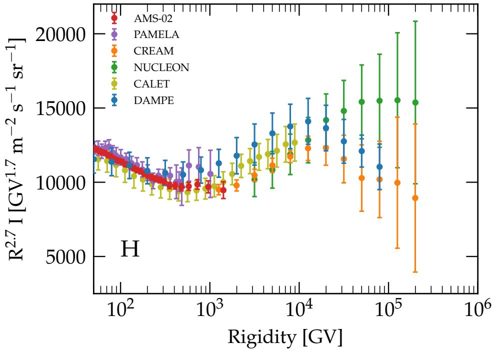


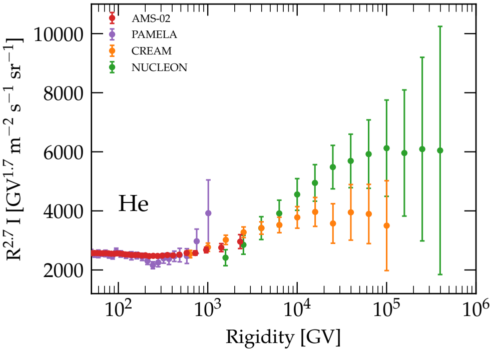
Key question!

Having evidence of a feature in the proton spectrum, how to distinguish if due to injection or propagation?

Galactic Cosmic Rays: novel features

PAMELA Coll., Science 2011; AMS-02 Coll., PRL 2015; CREAM Coll., ApJ 2017; NUCLEON Coll., JETP 2018; DAMPE Coll., Science 2019





- Spectra of protons and helium are not a single power law below the knee $\rightarrow$ some physics kicking in?
- The hardening at $R = p/Z \sim 300 - 400$ GV is well established since first observation by PAMELA
- AMS-02 confirmed the same break for almost all nuclei
- The softening at $R = p/Z \sim 10$ TV is observed by different experiments, first strong evidence in DAMPE

The cosmic ray density

- Cosmic rays come from all directions in outer space over large energy intervals.
- The number of particles in volume element $d^3 r$ about $\vec{r}$ and in the momentum interval $d^3 p$ about $\vec{p}$ is given by

$$dn = F(\vec{r}, \vec{p}, t)\, d^3 r\, d^3 p$$

  with F the distribution function.
- Expanding in spherical coordinates $d^3 p$:

$$dn = F(\vec{r}, p, t)\, d^3 r\, p^2\, dp d\Omega$$

- Typically we are not able to measure F but only averages over momentum space, thereby we conveniently introduce the phase-space distribution function as:

$$f(\vec{r}, p, t) = \frac{1}{4\pi} \int_\Omega F(\vec{r}, p, t) d\Omega$$

- Correspondingly, the number of particles dN in $d^3\, r$ and in $(p, p + dp)$ (independent of direction of $\vec{p}$) is:

$$dN = \int_\Omega d\Omega\, F(\vec{r}, p, t)\, d^3 r\, p^2 dp = 4\pi p^2 f(\vec{r}, p, t)\, d^3 r\, dp$$

Description of transport of nuclei

- For nuclei of mass A, it is customary to introduce the **intensity** (number of particles per unit surface time solid angle and energy) as a function of the **kinetic energy per nucleon** T:

$$I_\alpha(T) dT = p^2 f_\alpha(p) v(p) dp \longrightarrow I_\alpha(T) = A p^2 f_\alpha(p)$$

! fragmentation preserves the energy per nucleon

- I is the quantity to be directly compared with measurements

- We explicit $q_{\rm SN} = 2 h_d \delta(z) q_\alpha(p)$ and $(\tau_\alpha^{\rm in})^{-1} = 2 h_d \delta(z) n_d v \sigma_\alpha$, thereby the transport equation becomes:

diffusion; spallation of nuclei α; injection of nuclei α; contribution to nuclei α from spallation of $\alpha' > \alpha$

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(T)}{\partial z}\right] + 2 h_d n_d v(T) \sigma_\alpha \delta(z) I_\alpha(T) = 2 A p^2 h_d q_{0,\alpha}(p) \delta(z) + \sum_{\alpha' > \alpha} 2 h_d n_d v(T) \sigma_{\alpha' \to \alpha} \delta(z) I_{\alpha'}(T)$$

The transport equation for primary Nuclei

- Formally similar to the equation for protons but with spallation taken into account:

$$-\frac{\partial}{\partial z}\left[D_\alpha \frac{\partial I_\alpha(T)}{\partial z}\right] + 2h_d n_d v(T)\sigma_\alpha \delta(z) I_\alpha(T) = 2Ap^2 h_d q_{0,\alpha}(p)\delta(z)$$

- The equation is solved in the same way:
  - first we consider the solution for $z \neq 0$ ($z > 0$ or $z < 0$)
  - then integrate around $z = 0$ between 0^- and 0^+
- It follows:

$$D_\alpha \frac{\partial I_\alpha}{\partial z} = \text{constant} \longrightarrow I_\alpha = I_{0,\alpha}\left(1 - \frac{z}{H}\right)$$

which we use to derive

$$-D_\alpha \frac{\partial I_\alpha}{\partial z}\bigg|_{z=0} = -h_d n_d v(T)\sigma_\alpha I_{0,\alpha} + Ap^2 h_d q_{0,\alpha}(p)$$

The transport equation for primary Nuclei

- The intensity of a primary nucleus of type α is

$$I_{0,\alpha}(T) = \frac{\frac{Ap^2 h_d q_{0,\alpha}(p)}{H} \frac{H^2}{D_\alpha}}{1 + \frac{\chi_\alpha(T)}{\hat{\chi}_\alpha}} = \frac{\frac{Ap^2 q_{0,\alpha}(p)}{n_d m_p v} \chi_\alpha(T)}{1 + \frac{\chi_\alpha(T)}{\hat{\chi}_\alpha}}$$

- Where the grammage traversed by nuclei of type α:

$$\chi_\alpha(T) = n_d \left(\frac{h}{H}\right) m_p v \frac{H^2}{D_\alpha} = \bar{n} m_p v \tau_{\text{esc}}(T)$$

- and the critical grammage (energy independent) is:

$$\hat{\chi}_\alpha = \frac{m_p}{\sigma_\alpha}$$

Relevant limits

diffusion dominated: for $\chi \ll \hat{\chi}$ the equilibrium spectrum is $I_0 \propto T^{-\gamma-\delta}$

spallation dominated: for $\chi \gg \hat{\chi}$ the equilibrium spectrum is $I_0 \propto T^{-\gamma}$

The transport equation for secondary Nuclei: secondary/primary ratio

- Let's work out a simple case with only Carbon as primary species $\alpha' = \mathrm{C}$, and Boron as secondary $\alpha = \mathrm{B}$:

destruction of B — production of B from C spallation

$$-\frac{\partial}{\partial z}\left[D_{\mathrm{B}}\frac{\partial I_{\mathrm{B}}(T)}{\partial z}\right] + 2h_d n_d v \sigma_{\mathrm{B}} \delta(z) I_{\mathrm{B}}(T) = 2h_d n_d v \sigma_{\mathrm{C}\to\mathrm{B}} \delta(z) I_{\mathrm{C}}(T)$$

- following the same approach as before (and assuming $\chi_{\mathrm{B}} \simeq \chi_{\mathrm{C}} \equiv \chi$):

$$I_{\mathrm{B},0}(T) = I_{\mathrm{C},0}(T)\frac{\chi(T)}{\hat{\chi}_{\mathrm{C}\to\mathrm{B}}}\left(1+\frac{\chi(T)}{\hat{\chi}_{\mathrm{B}}}\right)^{-1}$$

- which reflects in the following B/C ratio:

$$\frac{\mathrm{B}}{\mathrm{C}} = \frac{\frac{\chi(T)}{\hat{\chi}_{\mathrm{C}\to\mathrm{B}}}}{1+\frac{\chi(T)}{\hat{\chi}_{\mathrm{B}}}}$$

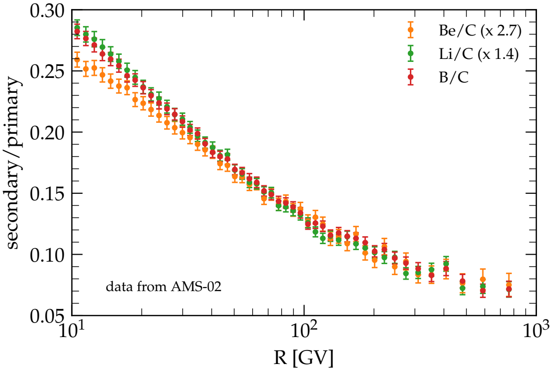
Relevant limits

diffusion dominated: for $\chi \ll \hat{\chi}$ the ratio is B/C $\propto \chi(T) \propto 1/D(T)$

spallation dominated: for $\chi \gg \hat{\chi}$ the ratio is B/C $\sim$ constant

The transport equation for secondary Nuclei: the diffusion coefficient slope

AMS-02 Coll., PRL 120 (2018)

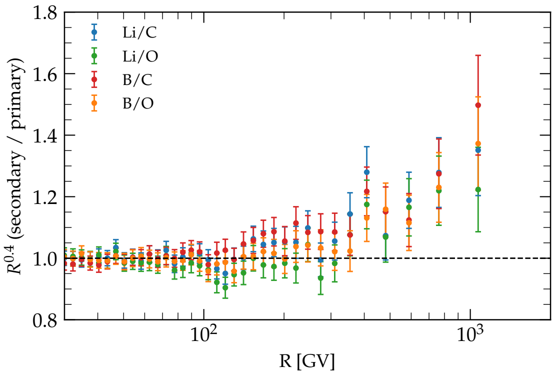


- Evidence of rigidity dependent grammage $\to$ high-energy particles spend less time in our Galaxy than low-energy ones (advection may play a role only at low energies)
- At $T \gtrsim 50$ GeV/n the B/C ratio scales as the grammage $\to$ we can measure the slope of $D(E)$ from the energy dependence of B/C.
- Notice however that B/C is sensitive only to the H/D ratio, remember:

$$\chi = n_d m_p v h \frac{H}{D_\alpha}$$

The transport equation for secondary Nuclei: the origin of the spectral feature

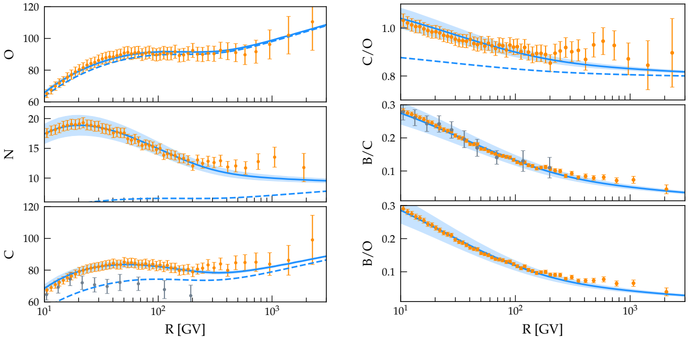
AMS-02 Coll., PRL 120 (2018)



The same feature detected in the primary spectra at ~ 300 GV is observed in the secondary/primary ratio which depends only on the grammage $\rightarrow$ **is it an effect during Galactic propagation or at the acceleration?**

Quick look at the data: The CNO element

Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), De La Torre Luque et al., JCAP 03 (2021), Schroer et al., PRD 103 (2021)

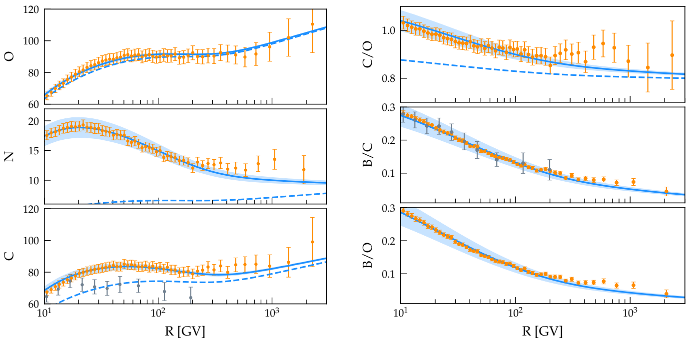


- We assume a phenomenological motivated $D(R)$ as a smoothly-broken power-law:

$$D(R) = \boxed{2v_A H} + \frac{\boxed{\beta D_0 (R/\text{GV})^{\delta}}}{\boxed{[1 + (R/R_b)^{\Delta\delta/s}]^s}}$$

Quick look at the data: The CNO element

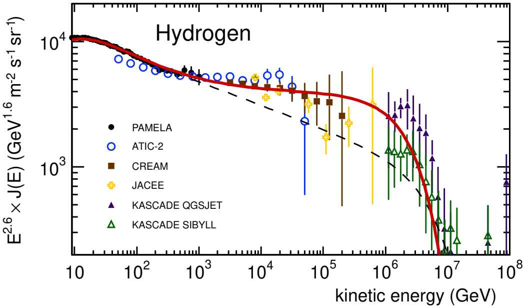
Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), De La Torre Luque et al., JCAP 03 (2021), Schroer et al., PRD 103 (2021)

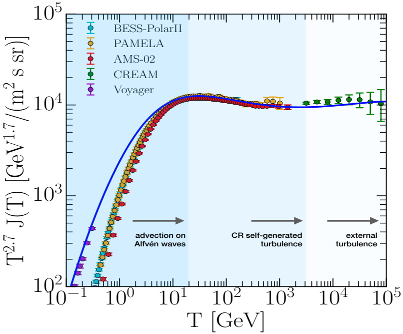


- by fitting primary and secondary/primary measurements we found:
  $\delta \sim 0.54$, $D_0/H \sim 0.5 \times 10^{28}$ cm/s^2/kpc, $\Delta\delta \sim 0.2$, $v_A \sim 5$ km/s
- All nuclei injected with $\gamma \sim 4.3$
- Shaded areas: uncertainty from cross sections (small for pure primary species as Oxygen).

Secondary-to-Primary Ratios and the Origin of the Hardening

Blasi+, PRL 2012; Tomassetti, A&A 2012





- At ~ 300 GV, a similar break is detected in the secondary-to-primary ratios $\rightarrow$ a change in CR transport within the Galaxy.
- Currently, two physical interpretations are proposed:
  - It marks the transition between the self-generation of turbulence by CRs themselves and the large-scale turbulence
  - The transition results from differing turbulence conditions in the disk and halo
- It remains unclear if these interpretations fully reproduce the sharpness of the observed feature

Decay of unstable isotopes

Problem!

B/C only gives the grammage $\propto H/D \rightarrow$ how to break the degeneracy?

- We now look at the ratio of **unstable** and **stable** species, as the lifetime introduces a **clock** breaking the degeneracy
- ^{10}Be is β^- **unstable** with an half-life $\tau_{1/2} \sim 1.39 \times 10^6$ years $\longrightarrow$ ^{10}B
- The transport equation for ^{10}Be is the first case we discuss where the source or loss term **is not in the form of a δ-function in z**:

destruction of Be — Be decay — production of Be from C spallation

$$-\frac{\partial}{\partial z}\left[D_{\text{Be}}\frac{\partial I_{\text{Be}}(T)}{\partial z}\right] + \frac{\mu v \sigma_{\text{Be}}}{m}\delta(z) I_{\text{Be}}(T) + \frac{I_{\text{Be}}(T)}{\gamma \tau_d} = \frac{\mu v \sigma_{\text{C}\to\text{Be}}}{m}\delta(z) I_{\text{C}}(T)$$

where $\mu = 2 h_d n_d m \sim 10^{-3}$ g/cm^2 is the disk surface density.

- ^{10}Be decays on a time scale $\gamma\tau_d$ that at some high-E becomes longer than $\tau_{\text{esc}} \longrightarrow$ stable
- ^{10}Be decays mainly into ^{10}B so that it changes the abundance of stable elements.

Decay of unstable isotopes

- Outside the disk $z \neq 0$ the transport equation becomes

$$-\frac{\partial}{\partial z}\left[D_{\text{Be}}\frac{\partial I_{\text{Be}}(T)}{\partial z}\right] + \frac{I_{\text{Be}}(T)}{\hat{\tau}_d} = 0$$

- the solution is in the form

$$I = A\mathrm{e}^{-\alpha z} + B\mathrm{e}^{\alpha z}$$

which implies $\alpha^{-1} \equiv \sqrt{D\hat{\tau}_d}$

- after imposing the proper boundary conditions we obtain (introducing $y \equiv \mathrm{e}^{\alpha H}$):

$$\frac{I_{\text{Be}}(z)}{I_{\text{Be},0}} = -\frac{y^2}{1-y^2}\mathrm{e}^{-\alpha z} + \frac{1}{1-y^2}\mathrm{e}^{\alpha z}$$

- the value of the distribution function at $z = 0$ can be obtained by the usual integration above/below disc:

$$-2D_{\text{Be}}\frac{\partial I_{\text{Be}}(T)}{\partial z}\bigg|_{0^+} + \frac{\mu v \sigma_{\text{Be}}}{m} I_{\text{Be},0}(T) = \frac{\mu v \sigma_{\text{C}\to\text{Be}}}{m} I_{\text{C},0}(T)$$

$$\longrightarrow I_{\text{Be},0}(T)\left[\frac{\sigma_{\text{Be}}}{m} - \frac{2D_{\text{Be}}}{\mu v H}\alpha H\frac{1+y^2}{1-y^2}\right] = \frac{\sigma_{\text{C}\to\text{Be}}}{m} I_{\text{C},0}(T)$$

Decay of unstable isotopes

- The transport equation in terms of X's becomes:

$$\frac{I_{\text{Be},0}}{I_{\text{C},0}}(T) = \frac{1}{\hat{X}_{\text{C}\to\text{Be}}} \left[\frac{1}{\hat{X}_{\text{Be}}} + \frac{1}{X'_{\text{Be}}(T)} \right]^{-1}$$

- At high energy: $\frac{H^2}{D_{\text{Be}}} \ll \hat{\tau}_d \longrightarrow \alpha H \to 0$

$$X'_{\text{Be}}(T) \longrightarrow X_{\text{Be}}(T)$$

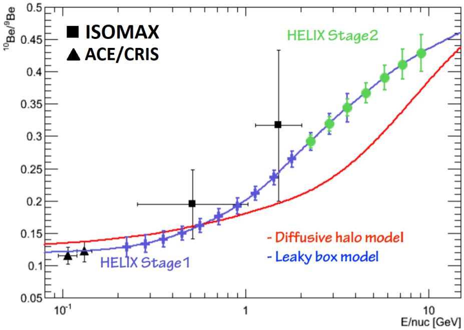
- At low energy: $\frac{H^2}{D_{\text{Be}}} \gg \hat{\tau}_d \longrightarrow \alpha H \to \infty$

$$X'_{\text{Be}}(T) \longrightarrow \frac{\mu v}{2} \sqrt{\frac{\hat{\tau}_d}{D_{\text{Be}}}}$$

$\longrightarrow$ independent on H!

- Still missing the additional contribution to B production by Be decay (small effect though).

Go HELIX go!

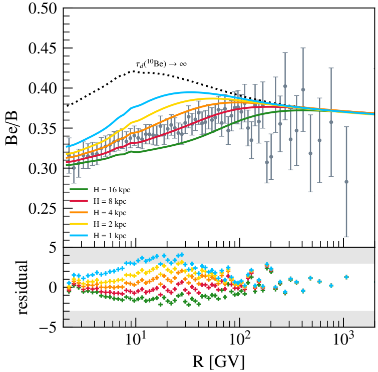


Homework!

Assuming the same production cross-section for Be10 and Be9 from parent Carbon $\longrightarrow$ use the HELIX expected value of Be10/Be9 to determine the confinement time at 10 GeV/n.

Quick look at the data: The Beryllium-over-Boron ratio and the Halo size

Evoli et al., PRD 101 (2020), Weinrich et al., A&A 639 (2020), Korsmeier & Cuoco, PRD 105 (2022), Maurin et al., arXiv:2203.07265



- Preference for large halos $H \gtrsim 5$ kpc
- Notice that H and $\tau_{\rm esc}$ are mutual corresponding

$$\tau_{\rm esc}(10\,\text{GV}) \sim \frac{H^2}{2D} \sim 100\,\text{Myr}\left(\frac{H}{5\,\text{kpc}}\right)\left(\frac{10^{28}\,\text{cm}^2/\text{s}/\text{kpc}}{D_0/H}\right)$$

Potential sources of galactic CRs

D. Ter Haar, Reviews of Modern Physics, 1950; Ginzburg & Syrovatskii, 1963

- The grammage is also a crucial piece of information to identify galactic CR sources.
- The galactic CR luminosity is:

from B/C

$$L_{\rm CR} \sim \frac{\epsilon_{\rm CR} V_{\rm MW}}{\tau_{\rm esc}} \sim \pi \epsilon_{\rm CR} R_d^2 \boxed{\frac{H}{D}} \sim 10^{41} \text{ erg/s}$$

where

- $\epsilon_{\rm CR} \sim 1$ eV/cm^3 is the local CR energy density
- $V_{\rm MW} = \pi R_d^2 2H$ is the Milky Way Volume (for CRs)
- $\tau_{\rm esc} \sim H^2/D$ is the escape time

- This is also the luminosity required (on a timescale of $\sim \tau_{\rm esc}$) to sustain the CR population.
- The SNe energy rate in our Galaxy:

$$\boxed{L_{\rm SN} = E_{\rm SN} R_{\rm SN} \sim 10^{42} \text{ erg/s} \sim 10 \times L_{\rm CR}}$$

- Galactic SNe provide the right energetics if $\sim 10\%$ efficiency in CR acceleration is achieved $\longrightarrow$ a mechanism able to transfer such an energy was discovered in the 70's (DSA).

Cosmic ray transport for the poor physicists

- Generic rule of thumb:

$$\text{Intensity} \sim \text{Injection Rate} \times \frac{\text{Relevant lifetime}}{\text{Relevant volume}}$$

- Primary species equilibrium spectrum:

$$I_p(T) \propto Q(T)\frac{\tau_{\rm esc}(T)}{H}$$

- Secondary stable species equilibrium spectrum:

$$I_s(T) \propto I_p(T)\sigma v n_{\rm d} h_d \frac{\tau_{\rm esc}(T)}{H}$$

- Secondary unstable(*) species equilibrium spectrum:

$$I_s^*(T) \propto I_p(T)\sigma v n_{\rm d} h_d \frac{\tau_{\rm d}(T)}{\sqrt{\tau_{\rm d}(T)D(T)}}$$

- Stable secondary over primary ratio:

$$\frac{I_s(T)}{I_p(T)} \propto \chi(T) \propto \frac{H}{D(T)}$$

- Unstable secondary over stable secondary ratio:

$$\frac{I_s^*(T)}{I_s(T)} \propto \frac{\sqrt{D(T)}}{H^2} \quad \leftarrow \text{break the degeneracy!}$$

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Charged particles in a ordered B-field

- In general the equation of motion of a charged particle in a electromagnetic fields

$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

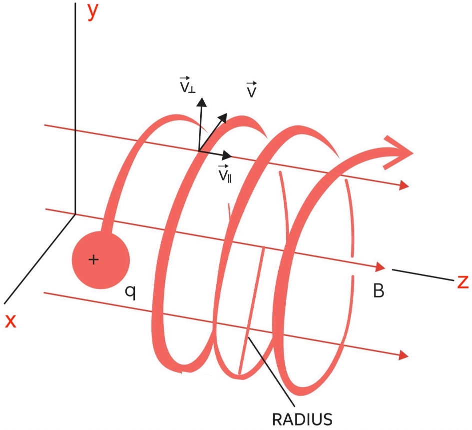
- Given the absence of regular electric fields $\vec{E} \to 0$ we will limit ourselves to the case where only $\vec{B}$ is present $\to$ as a consequence the particle energy γ cannot change
- We specify the eq. of motion for a regular magnetic field B_0 oriented along the z-axis

$$\begin{aligned} m\gamma \frac{dv_x}{dt} &= q\frac{v_y}{c}\mathrm{B}_0 \\ m\gamma \frac{dv_y}{dt} &= -q\frac{v_x}{c}\mathrm{B}_0 \\ \frac{dv_z}{dt} &= 0 \longrightarrow v_z \equiv v_{||} = \text{constant} \end{aligned}$$

- Combining the first two:

$$m\gamma \frac{d^2 v_{x,y}}{dt^2} = -\left(\frac{q\mathrm{B}_0}{mc\gamma}\right)^2 v_{x,y} \equiv -\Omega^2 v_{x,y}$$

Charged particles in a ordered B-field



$$\Omega \equiv \frac{qB_0}{mc\gamma} = \frac{v}{r_\mathrm{L}}$$ Gyration frequency, $$\mu \equiv \cos\theta = \frac{v_\parallel}{v}$$ Pitch angle

Charged particles in a ordered B-field

- The solution can be written as

$$\begin{aligned} v_x(t) &= A\cos(\Omega t) + B\sin(\Omega t) \\ v_y(t) &= -A\sin(\Omega t) + B\cos(\Omega t) \end{aligned}$$

- where A and B satisfy the initial conditions that

$$\begin{aligned} v_x(t=0) &= A \equiv v_\perp \cos(\phi) \\ v_y(t=0) &= B \equiv v_\perp \sin(\phi) \end{aligned}$$

- hence

$$\begin{aligned} v_x(t) &= v_\perp [\cos(\phi)\cos(\Omega t) + \sin(\phi)\sin(\Omega t)] = v_\perp \cos(\phi - \Omega t) \\ v_y(t) &= v_\perp [-\cos(\phi)\sin(\Omega t) + \sin(\phi)\cos(\Omega t)] = v_\perp \sin(\phi - \Omega t) \end{aligned}$$

- The unperturbed motion of the particle is periodic in the XY plane and rectilinear uniform in the z direction where

$$v_\parallel = v\mu = \text{constant} \to \mu = \text{constant}$$

and the equation of motion along z is simply

$$\boxed{z = v\mu t}$$

Charged particles in a ordered + random B-field

- For simplicity let's consider the case of a perturbation that only propagates along the ordered magnetic field $\vec{B} = \mathrm{B}_0 \hat{z}$ and only having components along x and y axes: $\delta\mathrm{B}_x,\ \delta\mathrm{B}_y$
- Is it still true that we can neglect induced electric fields?
- The equation of motion of the particle is

$$m\gamma \frac{d\vec{v}}{dt} = \frac{q}{c} \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \delta\mathrm{B}_x & \delta\mathrm{B}_y & \mathrm{B}_0 \end{pmatrix} \overset{\delta\mathrm{B} \ll \mathrm{B}_0}{\simeq} \frac{q}{c} \begin{pmatrix} v_y \mathrm{B}_0 \\ -v_x \mathrm{B}_0 \\ v_x \delta\mathrm{B}_y - v_y \delta\mathrm{B}_x \end{pmatrix}$$

- It follows that the **perturbed** motion along z becomes

$$m\gamma \frac{dv_z}{dt} = \frac{q}{c} \left[v_x(t) \delta\mathrm{B}_y - v_y(t) \delta\mathrm{B}_x \right]$$

as a consequence now the pitch angle changes with time

$$\rightarrow m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \left[\cos(\phi - \Omega t) \delta\mathrm{B}_y - \sin(\phi - \Omega t) \delta\mathrm{B}_x \right]$$

Charged particles in a ordered + random B-field

- Let's assume that the perturbed field is circularly polarized (and for simplicity we ignore the phase):

$$\delta B_y = \delta B \exp\left[i(kz - \omega t)\right]$$
$$\delta B_x = \pm i \delta B_y$$

- Taking the real part gives

$$\delta B_y = \delta B \cos(kz - \omega t)$$
$$\delta B_x = \mp \delta B \sin(kz - \omega t)$$

- thereby

$$m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \delta B \left[\cos(\phi - \Omega t)\cos(kz - \omega t) \pm \sin(\phi - \Omega t)\sin(kz - \omega t)\right]$$

or

$$m\gamma v \frac{d\mu}{dt} = \frac{q}{c} v_\perp \delta B \cos(\phi - \Omega t \mp kz \pm \omega t)$$

Charged particles in a ordered + random B-field

- For Alfvén waves the dispersion relation holds $\omega = k v_\mathrm{A}$ where v_A is the Alfvén velocity, as a consequence

$$\frac{kz}{\omega t} \simeq \frac{k v \mu t}{k v_\mathrm{A} t} \sim \frac{v \mu}{v_\mathrm{A}} \gg 1$$

  unless $\mu \ll v_\mathrm{A}/v$.

- We are allowed to neglect the term ωt with respect to kz. More formally, this is equivalent to choose the reference system in which waves are stationary. This implies that in this frame there is no electric field associated with the waves.

- Finally, using again that for unperturbed orbit $z = v \mu t$

$$\frac{d\mu}{dt} = \frac{q\mathrm{B}_0}{mc\gamma} \frac{v_\perp}{v} \frac{\delta \mathrm{B}}{\mathrm{B}_0} \cos\left[\phi + (\Omega \pm k v \mu) t\right]$$

  or

$$\boxed{\frac{d\mu}{dt} = \frac{q\mathrm{B}_0}{mc\gamma} (1 - \mu^2)^{\frac{1}{2}} \frac{\delta \mathrm{B}}{\mathrm{B}_0} \cos\left[\phi + (\Omega \pm k v \mu) t\right]}$$

Charged particles in a ordered + random B-field

- Once averaged over a long period of time, the **mean value** of the displacement in the cosine of the pitch angle must vanish

$$\langle \Delta\mu \rangle = \int_0^{\Delta t} dt \frac{d\mu}{dt} \to 0$$

- However, its variance does not

$$\langle \Delta\mu\Delta\mu \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\Delta t} dt \frac{d\mu}{dt}(t) \int_0^{\Delta t} dt' \frac{d\mu}{dt}(t')$$

- Notice however that

$$\int_0^{\Delta t} dt \int_0^{\Delta t} dt' \cos\left[(\Omega \pm kv\mu)t\right] \cos\left[(\Omega \pm kv\mu)t'\right]$$

$$\overset{\Delta t \gg t,t'}{\simeq} \frac{1}{2} \int_0^{\Delta t} dt \cos\left[(\Omega \pm kv\mu)t\right] \int_{-\infty}^{\infty} dt' \cos\left[(\Omega \pm kv\mu)t'\right]$$

$$= \pi \int_0^{\Delta t} dt \cos\left[(\Omega \pm kv\mu)t\right] \delta\left(\Omega \pm kv\mu\right) = \Delta t\, \pi\delta\left(\Omega \pm kv\mu\right)$$

- Finally,

$$\langle \Delta\mu\Delta\mu \rangle = \pi\Omega^2 \frac{1-\mu^2}{2} \Delta t \left(\frac{\delta B}{B_0}\right)^2 \delta\left(\Omega \pm kv\mu\right)$$

Pitch angle diffusion

$$\langle \Delta\mu \rangle = 0\,, \quad \langle \Delta\mu\Delta\mu \rangle = \pi\Omega^2 \frac{1-\mu^2}{2} \Delta t \left(\frac{\delta B}{B_0}\right)^2 \delta\left(\Omega \pm kv\mu\right)$$

- The mean value of the square of the pitch angle variation is proportional to the time lapse $\rightarrow$ diffusion
- This is true only when the resonance condition is fulfilled:

$$\Omega \pm kv\mu = 0 \longrightarrow k = k_{\rm res} \equiv \frac{\Omega}{v\mu}$$

- The scattering depends on the power available at the resonant scale

$$\frac{\delta B^2(k)}{B_0^2} = \frac{\text{energy density in the turbulent field}}{\text{energy density in the regular field}} \equiv W(k)dk \equiv \mathcal{F}(k)$$

- What do you expect happening when $\mu \to 0$?

Diffusion coefficient in the presence of a spectrum of waves

- We can introduce now a diffusion coefficient in μ as

$$D_{\mu\mu}(k) = \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t}\right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)\frac{\delta \mathrm{B}^2(k_{\mathrm{res}})}{\mathrm{B}_0^2} k_{\mathrm{res}}\delta\left(k - k_{\mathrm{res}}\right)$$

- which in the presence of a spectrum of waves becomes

$$D_{\mu\mu} = \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t}\right\rangle = \frac{\pi}{2}\Omega(1-\mu^2)\int dk \frac{\delta \mathrm{B}^2(k)}{\mathrm{B}_0^2} k\delta\left(k - k_{\mathrm{res}}\right)$$

- in terms of the dimensionless power in perturbations:

$$\boxed{D_{\mu\mu} = \frac{\pi}{2}\Omega(1-\mu^2)\mathcal{F}(k_{\mathrm{res}})}$$

- and the timescale necessary for deflection by $\mathcal{O}(\frac{\pi}{2})$ may be estimated as

$$\tau_{\mathrm{D}} \sim \frac{1}{D_{\mu\mu}} \simeq \frac{1}{\Omega\mathcal{F}(k_{\mathrm{res}})}$$

notice that being Ω the gyration frequency in the unperturbed field, this expression is telling us that the particles must perform many gyrations in order to get a deflection by order unity.

Spatial diffusion

- The diffusion coefficient of particles in space, following the general definition is

$$D_{zz}(p) \simeq \frac{1}{3} v \lambda(p) \simeq \frac{1}{3} v v \tau_{\rm D}(p) = \frac{1}{3} v \frac{r_{\rm L}}{\mathcal{F}(k_{\rm res})}$$

- or in terms of the Bohm diffusion coefficient $D_{\rm B} = \frac{1}{3} v r_{\rm L}$

$$D_{zz}(p) \simeq D_{\rm B}(p) \frac{1}{\mathcal{F}(k_{\rm res})}$$

What B/C does imply on scattering micro-physics?

- By reproducing local measurements we obtained:

from B/C — from Be/B

$$\boxed{D(\mathrm{GV})/H \simeq 10^{29}\ \mathrm{cm}^2/\mathrm{s}/\mathrm{kpc}} + \boxed{H \simeq 5\ \mathrm{kpc}} \to \boxed{D(\mathrm{GV}) \simeq 5 \times 10^{29}\ \mathrm{cm}^2/\mathrm{s}}$$

- In terms of a diffusion coefficient:

$$D(E) = \frac{1}{3} r_{\mathrm{L}}(E) v \frac{1}{\mathcal{F}(k_{\mathrm{res}})} = \frac{1}{3} v \lambda_{\mathrm{diff}}(E) \quad \text{where} \quad k_{\mathrm{res}} \sim \frac{1}{r_{\mathrm{L}}(E)}$$

- implying that at $\sim$ GV:

$$\lambda_{\mathrm{diff}} \simeq \mathrm{pc}$$

remember this is (on average) how much a GV particle has to travel before to deflect by 90°

- the turbulence level required to do so

$$\boxed{r_{\mathrm{L}}(\mathrm{GV}) \simeq 10^{12}\ \mathrm{cm} \to \mathcal{F}(k) \simeq \frac{r_{\mathrm{L}} c}{3 D_0} \simeq 6 \times 10^{-7} = \left(\frac{\delta B}{B_0}\right)^2_{k_{\mathrm{res}}}}$$

notice that we prove a posteriori the validity of the perturbative (QLT) approach.

Resonance condition

- The spatial diffusion coefficient in QLT is given by

$$D_{zz}(p) \simeq D_{\rm B}(p) \frac{1}{\mathcal{F}(k_{\rm res})}$$

- where the resonance condition holds ($\longrightarrow$ blackboard!)

$$\underbrace{k_{\rm res}}_{\text{wave}} = \underbrace{\frac{1}{\mu r_{\rm L}}}_{\text{particle}}$$

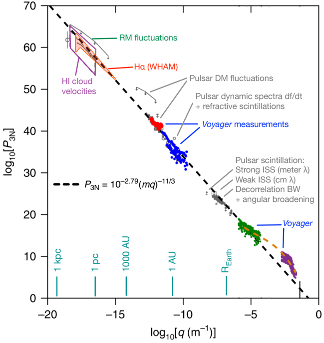
- Numerically:

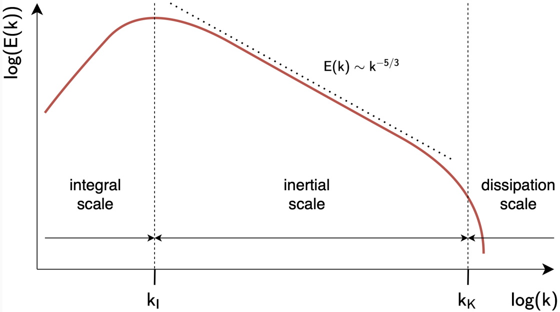
$$r_{\rm L} \simeq 10\,\text{parsec} \left(\frac{E}{\text{PeV}}\right) \left(\frac{B}{\mu\text{G}}\right)^{-1}$$

- What happens when the particle Larmor radius ($\longrightarrow$ energy) exceeds the larger wavelength in the ISM?

The CR knee!

Resonance condition





$$D_{zz}(p) \simeq D_{\rm B}(p)\frac{1}{\mathcal{F}(k_{\rm res})} \to D_{zz} \sim E^{2-\beta}$$

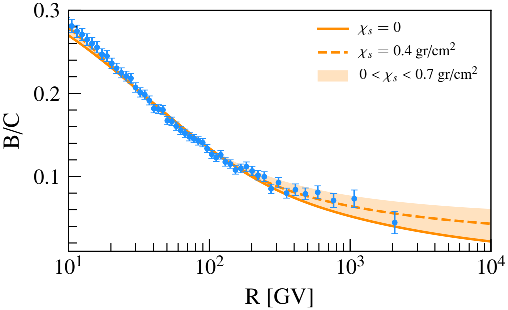
We found the connection between phenomenology δ and the micro-physics world β!

Another example of “Little things affect Big things”

Transport ($\sim 10^{22}$ cm) $\longrightarrow$ mean free path ($\sim 10^{18}$ cm) $\longrightarrow$ waves lenght ($\sim 10^{13}$ cm)

Such a tiny perturbation at the scale of the Solar System stretches the transport time in the Galaxy from kyrs’ to 100 Million of years!

Additional effects not included in this picture



- Second-order Fermi acceleration in the ISM [Ptuskin et al., 2006, ApJ 642; Drury & Strong, 2017, A&A 597]
- Shock re-acceleration of secondary nuclei [Blasi, 2017, MNRAS 471; Bresci et al., 2019, MNRAS 488]
- Grammage at the sources [D'Angelo et al., 2016, PRD 94; Nava et al., 2016, MNRAS 461; Jacobs et al., 2022, JCAP 05]
- Secondary production at the sources [Blasi, 2009, PRL 103; Mertsch & Sarkar, 2014, PRD 90]
- ...

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Non-linear cosmic ray transport

Skilling, ApJ 1971; Kulsrud & Cesarsky, ApJL 1971; Wentzel, ARAA 1974

- Spatial diffusion tends to reduce the CR momentum forcing them to move at the wave speed v_A [Kulsrud's book (2004)]:

$$\frac{dP_{\rm CR}}{dt} = -\frac{n_{\rm CR} m (v_D - v_A)}{\tau} \longrightarrow \text{Waves}$$

- If CR stream faster than the waves ($v_D > v_A$) the net effect of diffusion is to make waves grow: this process is known as self-generation of waves (notice that self-generated waves are such $k \sim r_L$)
- Waves are amplified by CRs through streaming instability:

$$\Gamma_{\rm CR} = \frac{16\pi^2}{3} \frac{v_A}{kW(k) B_0^2} \left[v(p) p^4 \frac{\partial f}{\partial z} \right] \propto \frac{P_{\rm CR}(>p)}{P_{\rm B}} \frac{v_A}{H} \frac{1}{kW(k)}$$

- and are damped by wave-wave interactions that lead the development of a turbulent cascade (NLLD):

$$\Gamma_{\rm NLLD} = (2c_k)^{-3/2} k v_A (kW)^{1/2}$$

What is the typical scale/energy up to which self-generated turbulence is dominant?

Non-linear cosmic ray transport

Blasi, Amato & Serpico, PRL, 2012

- Transition occurs at scale where external turbulence equals in energy density the self-generated turbulence:

$$W_{\rm ext}(k_{\rm tr}) = W_{\rm CR}(k_{\rm tr})$$

  where $W_{\rm CR}$ corresponds to $\Gamma_{\rm CR} = \Gamma_{\rm NLLD}$

- The normalization of $W_{\rm ext}$ is set to reproduce the CR flux much above the break:

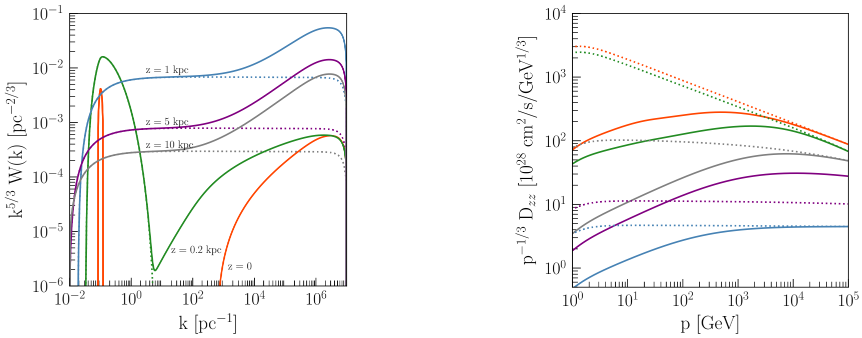
$$E_{\rm tr} = 228\,{\rm GeV} \left(\frac{R_{d,10}^2 H_3^{-1/3}}{\epsilon_{0.1} E_{51} \mathcal{R}_{30}} \right)^{3/2(\gamma_p - 4)} B_{0,\mu}^{(2\gamma_p - 5)/2(\gamma_p - 4)}$$

- Applying QLT it follows:

$$D_{\rm sg}(1\,{\rm GV}) \sim \frac{c r_L}{3} \frac{1}{k W_{\rm CR}(k)} \sim 10^{28} {\rm cm}^2 {\rm s}^{-1}$$

Non-linear cosmic ray transport: diffusion coefficient

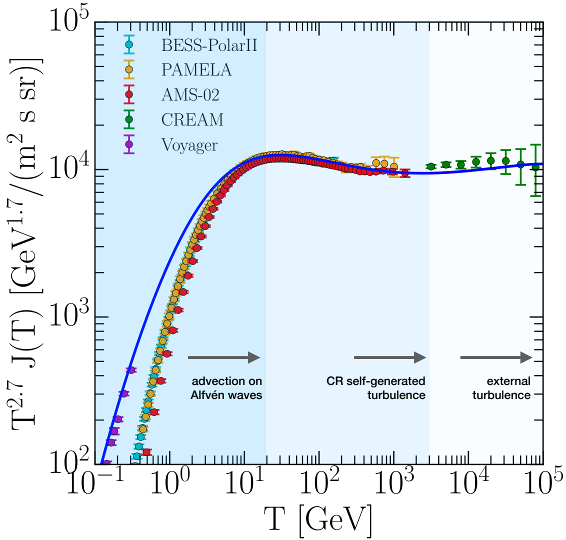
Evoli, Blasi, Morlino & Aloisio, PRL 2018



- Turbulence spectrum (left) and diffusion coefficient (right) without (dotted) and with (solid) CR self-generated waves at different distances from Galactic plane
- The wave advection originates the turbulent halo at a distance $\tau_{\rm cascade} = \tau_{\rm adv} \to z_{\rm H} \sim \mathcal{O}(\rm kpc)$
- $D(p,z)$ is now an output of the model

Non-linear cosmic ray transport: a global picture

Evoli, Blasi, Morlino & Aloisio, PRL 2018



- Pre-existing waves (Kolmogorov) dominates above the break.
- Self-generated turbulence between 1-100 GeV.
- Voyager data are reproduced with no additional breaks (single injection slope), but due to advection with self-generated waves (+ ionization losses).
- H is not predetermined here.
- None of these effects were included in the numerical simulations of CR transport before.

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A quick look to lepton energy losses

- The main difference with respect to the nuclei case is that leptons are very prone to radiative energy losses
- High-energy leptons lose energy predominantly for

  synchrotron emission on Galactic magnetic field

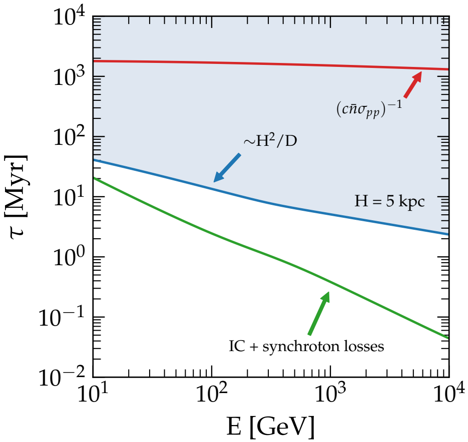
  inverse Compton scattering on Galactic radiation fields (CMB, IR, optical...)
- We limit our model to the Thomson limit, ignoring the corrections to the γ-e$^-$ cross-section due to the Klein-Nishina effect (= ignoring the electron recoil)
- the energy loss rate reads

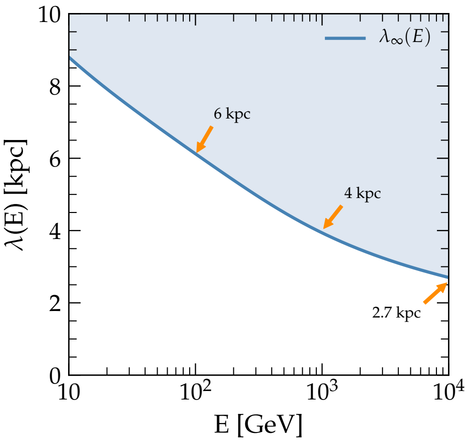
$$\left|\frac{dE}{dt}\right| = \frac{4}{3}\sigma_{\rm T} c \gamma^2 \beta^2 (\mathcal{U}_\gamma + \mathcal{U}_{\rm B}) \equiv b_0 \left(\frac{E}{10\ \text{GeV}}\right)^2$$

  where $\mathcal{U}_\gamma$ is the energy density in soft photons (IC) and $\mathcal{U}_{\rm B} = \frac{B^2}{8\pi}$ in magnetic field (synchrotron)
- in Galactic environments $\mathcal{U}_i \sim \mathcal{O}(0.1 - 1\ \text{eV/cm}^3) \to b_0 \sim 10^{-14} \left(\frac{\mathcal{U}_\gamma + \mathcal{U}_{\rm B}}{\text{eV/cm}^3}\right) \left(\frac{E}{10\ \text{GeV}}\right)^2$ GeV/s
- the energy loss time is a decreasing function with energy:

$$\tau_{\rm loss} \simeq \frac{E}{-\frac{dE}{dt}} \sim 3\ \text{Myr} \left(\frac{E}{10\ \text{GeV}}\right)^{-1}$$

How does it compare with the CR escape time in Galaxy?





- Leptons lose their energy mainly by IC with the interstellar radiation fields (ISRFs) or synchrotron emission
- Milky Way is a very inefficient calorimeter for nuclei and a perfect calorimeter for leptons
- Translate losses into propagation scale: $\lambda \sim \sqrt{D(E)\tau_{\rm loss}} \to$ horizon

Transport of leptons

- The transport equation to deal with is

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f_e}{\partial z}\right] = q_e(p)\delta(z) - \frac{1}{p^2}\frac{\partial}{\partial p}\left[\dot{p}p^2 f_e\right]$$

- It is convenient to approximate the loss terms as a catastrophic loss term

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f_e}{\partial z}\right] = q_e(p)\delta(z) - \frac{f_e}{\tau_{\rm loss}}$$

that can be solved similarly to Be since the energy losses are effective in all the propagation volume.

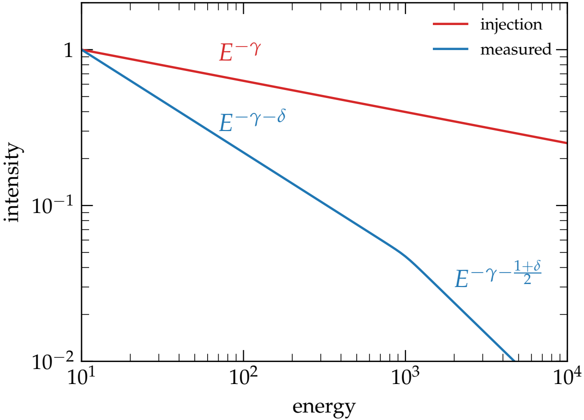
- Therefore, in the low energy limit where losses are weak, $\tau_{\rm loss} \gg \tau_{\rm esc}$:

$$f_{e,0}(E) = \frac{q_{e,0}(E)\mathcal{R}_{\rm SN}}{2\pi R_d^2}\frac{\tau_{\rm esc}}{H} \sim E^{-\gamma-\delta}$$

- In the high energy limit where losses dominate transport, $\tau_{\rm loss} \ll \tau_{\rm esc}$:

$$f_{e,0}(E) = \frac{q_{e,0}(E)\mathcal{R}_{\rm SN}}{2\pi R_d^2}\frac{\tau_{\rm loss}}{\sqrt{D\tau_{\rm loss}}} \sim E^{-\gamma-\frac{\delta+1}{2}}$$

Transport of leptons



For fiducial values of CR transport in the Milky Way the transition between the two regimes is at $\lesssim 10$ GeV

A quick application to the positron fraction

- Secondary positrons are produced through $pp \to \pi^\pm + \dots$ and tipically the energy of the secondary positron is a fraction $\xi \sim \mathcal{O}(10\%)$ of the parent proton energy E_p:

$$E_{e^+} \simeq \xi E_p \quad \leftarrow \text{inelasticity}$$

- The rate of positron e^+ production in the ISM is then

$$q_{e^+}(E)dE = n_p(E_p)dE_p n_d \sigma_{\rm pp} c 2 h_d \delta(z)$$

- Applying standard solutions we approach the usual limits, when losses are unimportant:

$$f_{e^+}(E) = n_p\left(\frac{E}{\xi}\right)\frac{2c\sigma_{\rm pp}n_d h_d}{\xi}\frac{H}{D(E)}$$

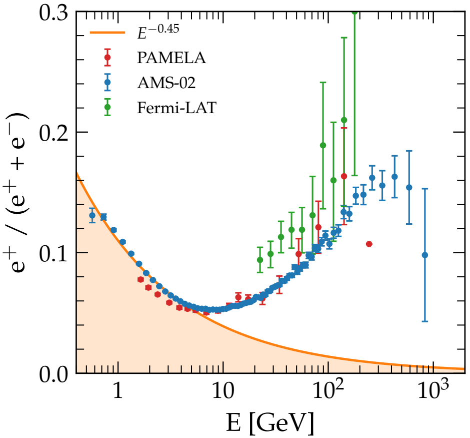
- while in the limit when losses dominate:

$$f_{e^+}(E) = n_p\left(\frac{E}{\xi}\right)\frac{2c\sigma_{\rm pp}n_d h_d}{\xi}\frac{\tau_{\rm loss}(E)}{\sqrt{\tau_{\rm loss}(E)D(E)}}$$

- as a consequence, in both cases (! homework):

$$\frac{f_{e^+}}{f_{e^-}}(E) = \frac{q_{p,0}(E/\xi)}{q_{e,0}(E)}\frac{1}{\xi}\frac{\chi(E/\xi)}{\hat{\chi}} \sim E^{-\gamma_p+\gamma_e-\delta}$$

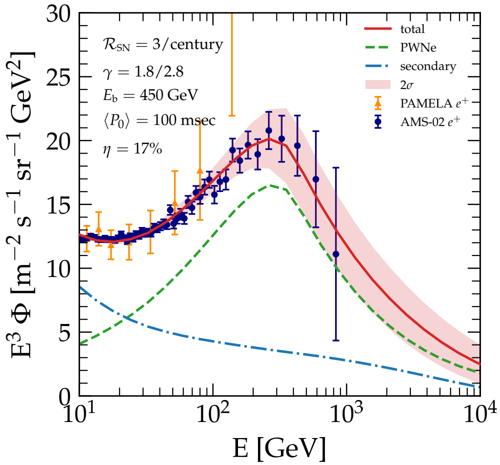
A quick application to the positron fraction



- Assuming $\gamma_p = \gamma_e \rightarrow$ positron fraction is a decreasing function with energy $\sim E^{-\delta}$
- To grow with energy must be $\gamma_e > \gamma_p + \delta$ unlikely!

Pulsars as positron galactic factories

Hooper+, JCAP 2009; Grasso+, APh 2009; Delahaye+, A&A 2010; Blasi & Amato 2011; Manconi+, PRD 2020; Evoli, Amato, Blasi & Aloisio, PRD 2021



$$Q_0(t) e^{-E/E_c(t)} \times \begin{cases} (E/E_b)^{-\gamma_L} & E < E_b \\ (E/E_b)^{-\gamma_H} & E \geq E_b \end{cases}$$

Shaded areas: 2-sigma fluctuations due to cosmic variance

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The Hillas Criterion

- The most effective particle accelerators are driven by electric fields (magnetic fields do not change the particle energy):

$$E_{\mathrm{max}} = q|\vec{E}|L$$

- Therefore, we need either large charges, strong electric fields, or large accelerators.
- However, in all astrophysical settings:

$$\vec{E} \to 0$$

- How do we overcome this? According to Faraday's law, a time-varying magnetic field can generate an electric field:

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

- Dimensionally:

$$\nabla \sim \frac{1}{L}, \quad \partial_t \sim \frac{1}{T} \quad \longrightarrow \quad E \sim \frac{L}{T}\frac{B}{c} \sim \frac{U}{cB}$$

- Therefore:

$$E_{\mathrm{max}} \sim \frac{qL}{cB}U$$

- In conclusion, we need fast magnetic fields!

The Hillas Criterion

- Notice that in order to get the most optimistic (but still plausible) result we assume $U \sim c$, in doing so, the criterion becomes

$$r_{\mathrm{L}} \sim L$$

  In order to be accelerated, a particle must be confined within its accelerator!

- Implicit assumption made in deriving the Hillas criterium: energy losses can be neglected. But this is, in general, not true!

Where to Find Fast Magnetic Fields?

- A supernova explosion marks the end of a massive star's life, releasing a vast amount of energy.
- When a star exhausts its nuclear fuel, its core collapses under gravity $\rightarrow$ Core Collapse SNe
- The core collapse converts gravitational potential energy into kinetic and thermal energy. For a typical massive star core (mass $\sim 1.4\,M_\odot$ and size R), this energy is approximately:

$$E_{\text{grav}} \sim \frac{GM^2}{R} \approx 10^{53}\,\text{erg}$$

- Most energy is initially released as neutrinos. A small fraction of this energy ($\sim 1\%$) is deposited into the outer layers, driving the explosion.
- The combined processes typically result in an explosion energy of about

$$E_{\text{kin}} \sim 10^{51}\,\text{erg}$$

Where to Find Fast Magnetic Fields?

- In the early stages, the mass of the ISM swept up by the shock is negligible compared to the ejecta mass ($\sim 1 M_\odot$):

$$\underbrace{M_{\rm ej}}_{\text{ejecta}} \gg \underbrace{M_{\rm sw}}_{\text{swept-up mass}}$$

- Therefore, in the so-called free expansion phase we have

$$E_{\rm SN} = \frac{1}{2} M_{\rm ej} v_{\rm sh}^2 \quad \rightarrow \quad v_{\rm sh} = \sqrt{\frac{2E_{\rm SN}}{M_{\rm ej}}} \approx 10^4 \left(\frac{M_{\rm ej}}{M_\odot} \right)^{-1/2} \text{ km/s}$$

- At later times, the situation reverses:

$$\underbrace{M_{\rm ej}}_{\text{ejecta}} \ll \underbrace{M_{\rm sw}}_{\text{swept-up mass}}$$

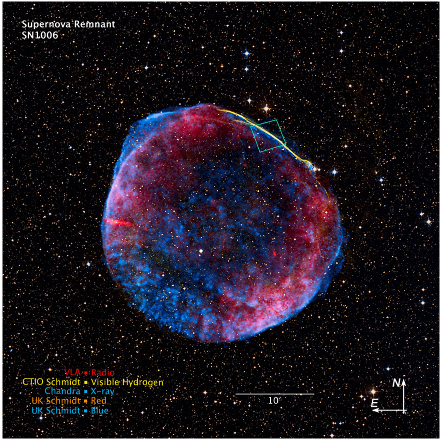
- Thus, in the Sedov-Taylor phase, the solution must depend on the ISM density $\rho_{\rm ISM}$ and not on $M_{\rm ej}$. By dimensional analysis:

$$v_{\rm sh} \sim \left(\frac{E_{\rm SN}}{\rho_{\rm ISM}} \right)^{1/5} t^{-3/5} \quad \rightarrow \quad v_{\rm sh} \approx 2 \times 10^3 \left(\frac{E_{\rm SN}}{10^{51} \text{ erg}} \right)^{1/5} \left(\frac{n_{\rm ISM}}{\text{cm}^{-3}} \right)^{-1/5} \left(\frac{t}{\text{kyr}} \right)^{-3/5} \text{ km/s}$$

- Applying the Hillas criterion to SNRs, we finally get:

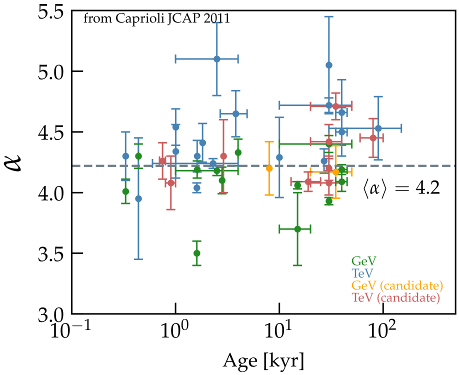
$$E_{\rm max} \sim 3 \times 10^{12} Z \left(\frac{B}{\mu \text{G}} \right) \overbrace{\left(\frac{U}{10^3 \text{ km/s}} \right)}^{\sim 1-10} \overbrace{\left(\frac{L}{\text{pc}} \right)}^{\sim 1-10} \text{ eV}$$

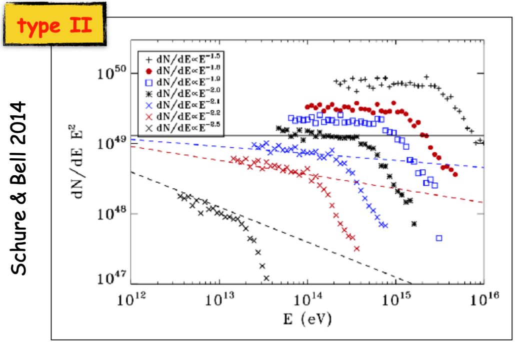
Large B fields are observed!



- Narrow X-ray synchrotron filaments hints B fields of $\mathcal{O}(100)$ ISM values: $L \simeq \sqrt{D_B \tau_l}$
- NON LINEAR! B is amplified during CR acceleration

So? What is the accelaration problem?





$$n_p \propto E^{-\gamma} \propto E^{-\alpha-\delta} \to \gamma(2.7) - \delta(0.4) = \alpha(?)$$

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Conclusions

- Cosmic Ray transport in the Galaxies is complex!
- The numerical diffusion models are certainly a big step forward, but don't forget are based on simple notions.
- The framework is still incomplete... time for new ideas!
- Need to look at all the observational constraints and model them simultaneously.
- These ideas and models can be straightforwardly applied to any astrophysical magnetized object.

Exercise

- Assume a power-law spectrum of turbulence in our Galaxy, given by:

$$W(k) \propto k^{-\alpha} \quad \text{for } k > k_0$$

  where $k_0 = 1\,\text{pc}^{-1}$ represents the turbulence correlation length. The value of α is $1/3$ for Kolmogorov turbulence and $1/2$ for Kraichnan turbulence.

- Given a grammage of 5 g/cm^2 at 10 GV, determine the ratio of the turbulent to regular magnetic field at the correlation-length scale, $\frac{\delta B}{B_0}(k_0)$, for both Kolmogorov and Kraichnan turbulence.
- The proton flux measured by AMS-02 at 10 GV is $21.55\,\text{GV}^{-1}\,\text{m}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}$. Using this data, calculate the efficiency of supernova energy conversion to cosmic rays, denoted by ξ.

Thank you!

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