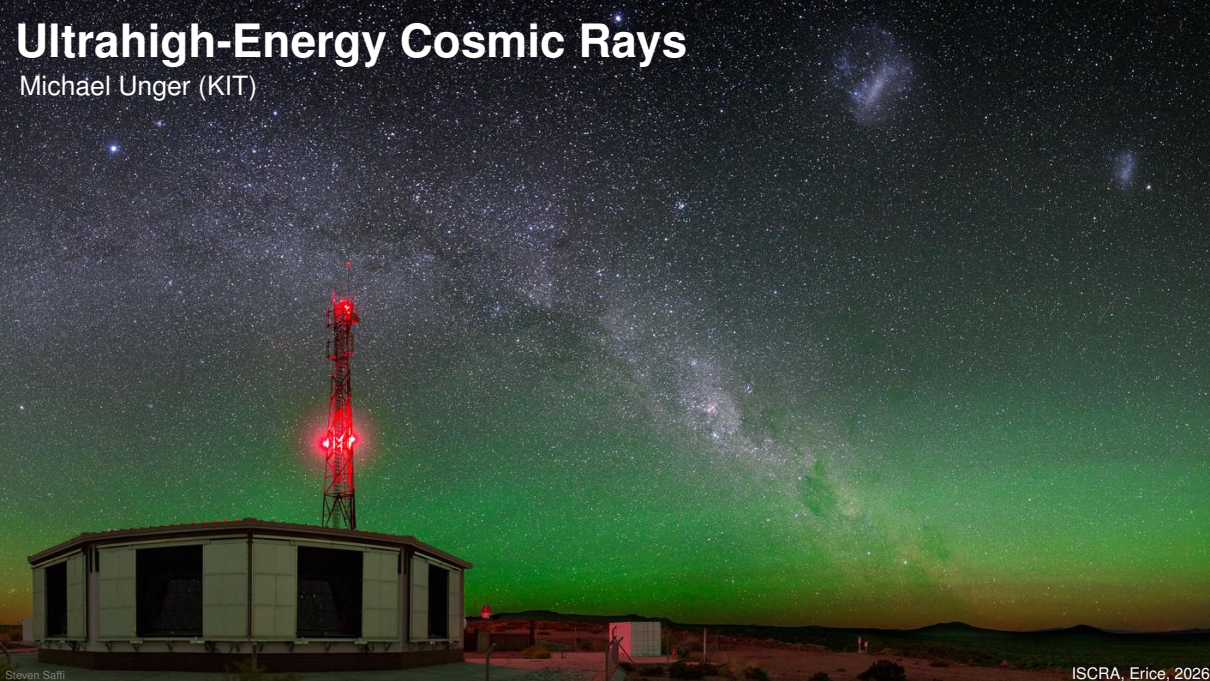
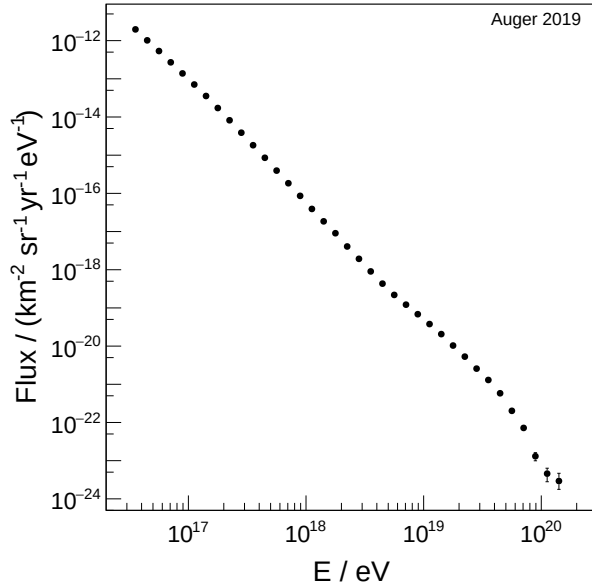


Ultrahigh-Energy Cosmic Rays

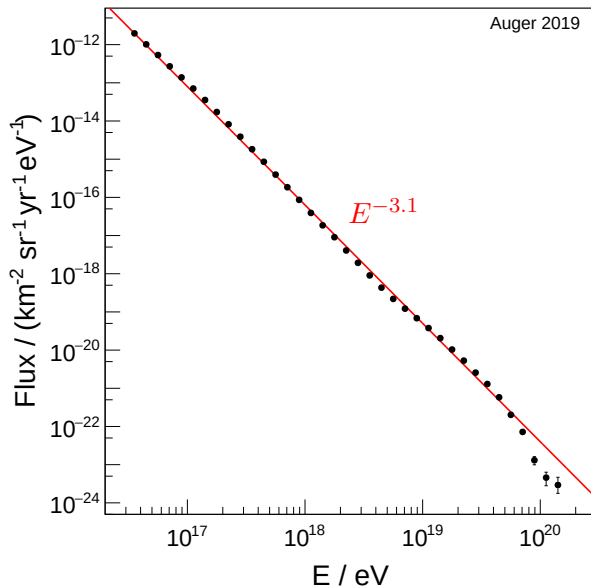
Michael Unger (KIT)



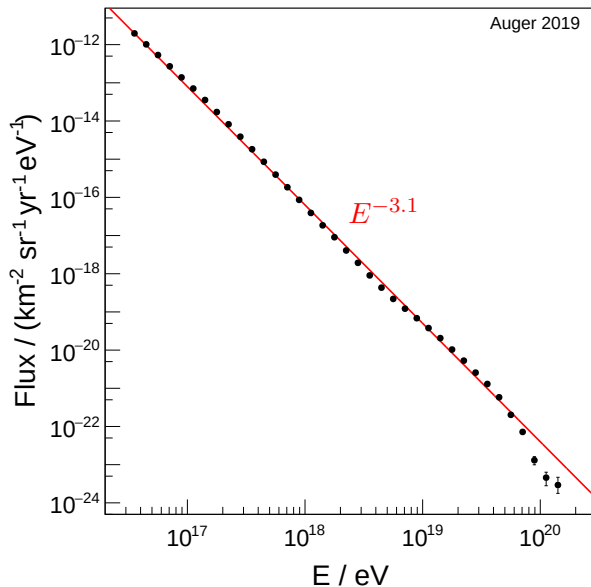
Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



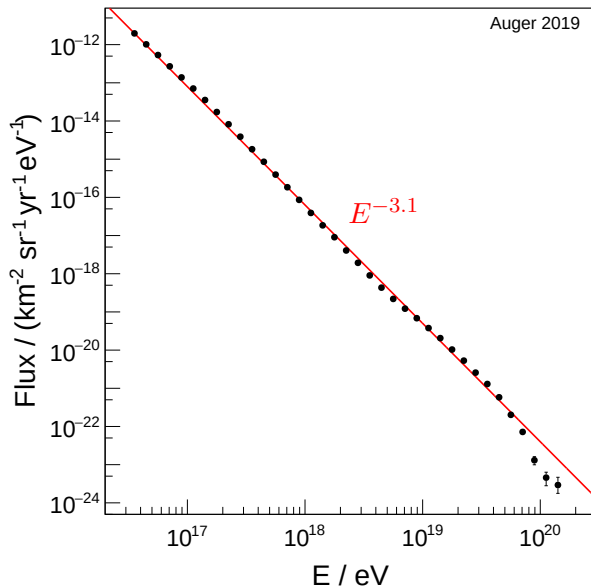
Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



$E_{\text{beam}}^{\text{LHC}} = 7 \times 10^{12} \text{ eV}$

←

Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



$E_{\text{kin}} \sim 4 \text{ TeV}$

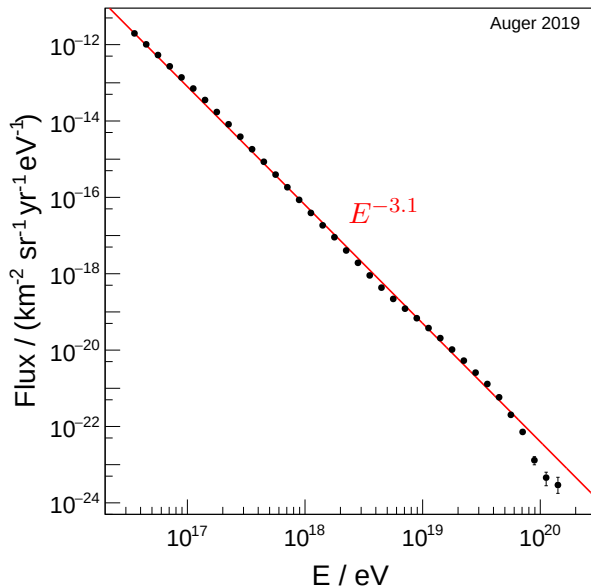
$E_{\text{beam}}^{\text{LHC}} = 7 \times 10^{12} \text{ eV}$

$\sim 20 \text{ J!}$

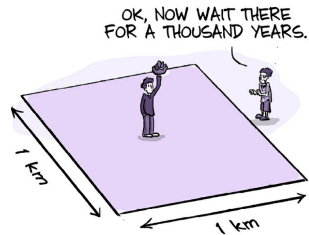


Serena Williams' 2nd serve

Energy Spectrum of Ultrahigh-Energy Cosmic Rays (UHECRs)



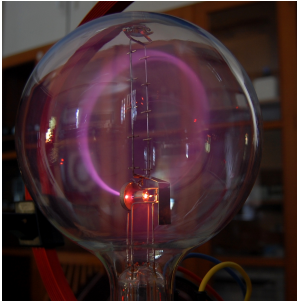
$E_{\text{beam}}^{\text{LHC}} = 7 \times 10^{12} \text{ eV}$



J. Cham & D. Whiteson "We have no idea", Penguin, 2018

ZeVatrons?

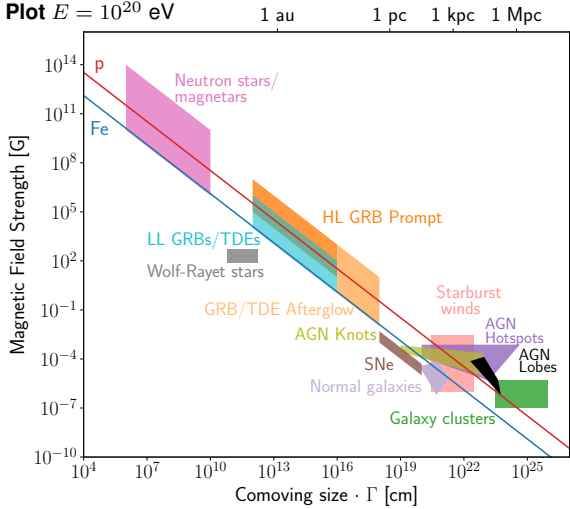
magnetic confinement in accelerator



using LHC magnets:

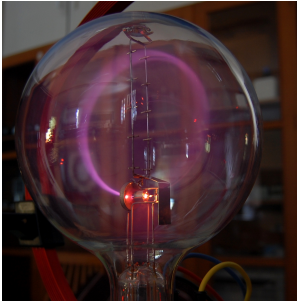


Hillas Plot $E = 10^{20}$ eV



ZeVatrons?

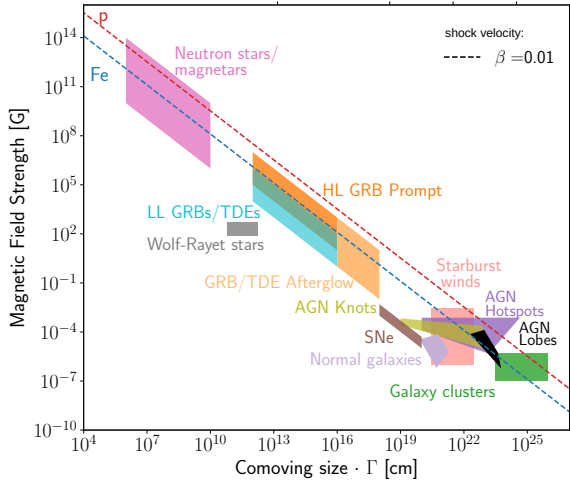
magnetic confinement in accelerator



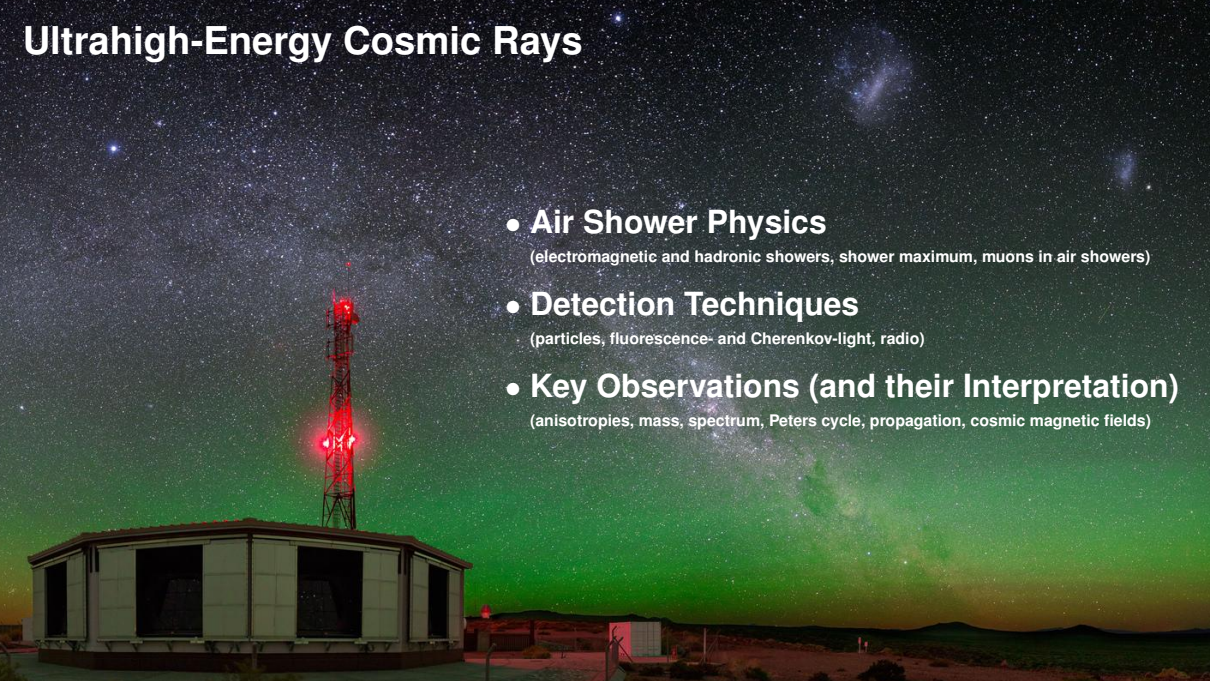
using LHC magnets:



Hillas Plot $E = 10^{20}$ eV 1 au 1 pc 1 kpc 1 Mpc



Ultra-high-Energy Cosmic Rays

A night sky with a green aurora borealis and a radio telescope structure. The sky is dark with many stars and a bright green aurora borealis. A radio telescope structure is visible on the left, with a red light at the top. The ground is dark and flat.

- **Air Shower Physics**

(electromagnetic and hadronic showers, shower maximum, muons in air showers)

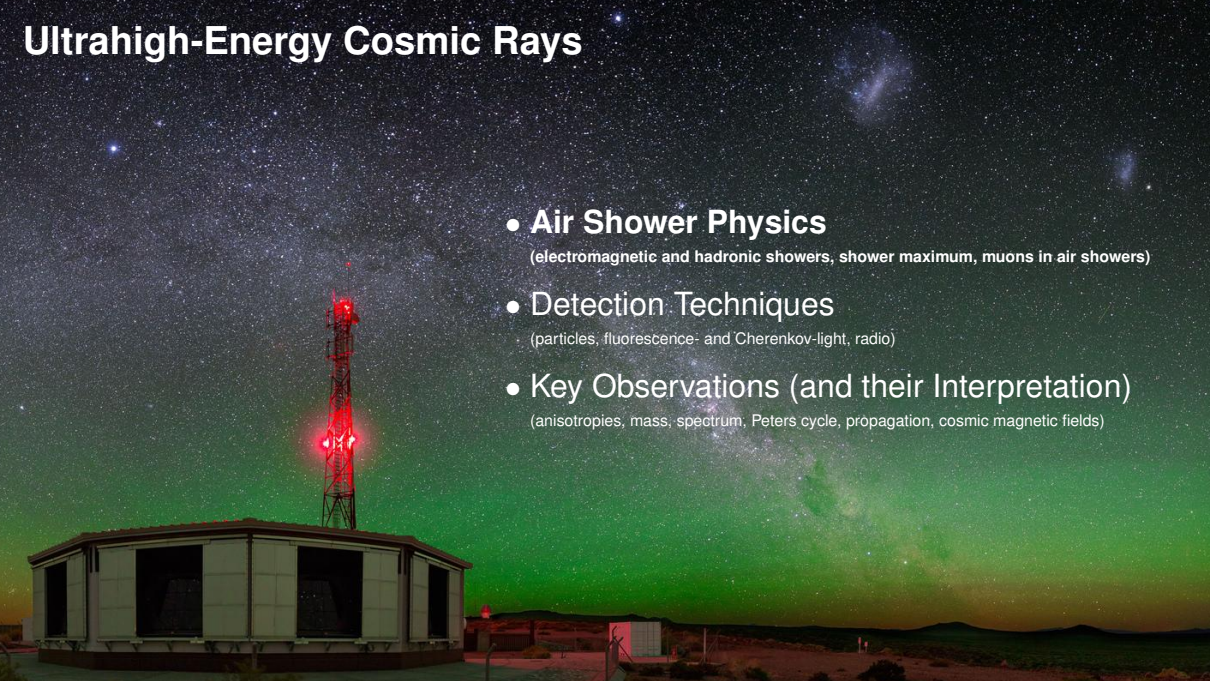
- **Detection Techniques**

(particles, fluorescence- and Cherenkov-light, radio)

- **Key Observations (and their Interpretation)**

(anisotropies, mass, spectrum, Peters cycle, propagation, cosmic magnetic fields)

Ultrahigh-Energy Cosmic Rays

A night sky with a green aurora borealis and a radio telescope structure. The sky is dark with many stars and a prominent green aurora borealis. In the foreground, there is a radio telescope structure with a tall tower and a large dish antenna. The tower is illuminated with red lights. The dish antenna is white and has a large opening. The background shows a desert landscape with some buildings and a fence.

- **Air Shower Physics**

(electromagnetic and hadronic showers, shower maximum, muons in air showers)

- **Detection Techniques**

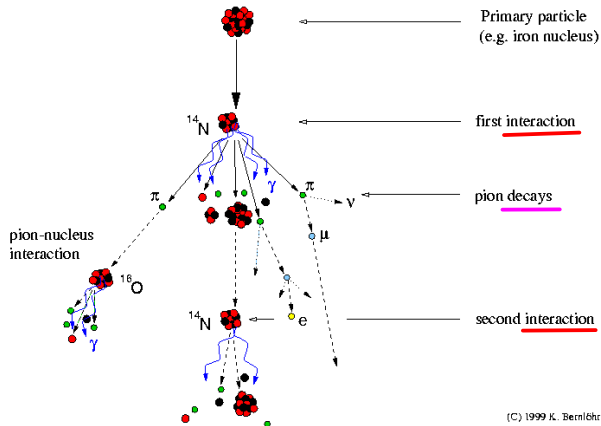
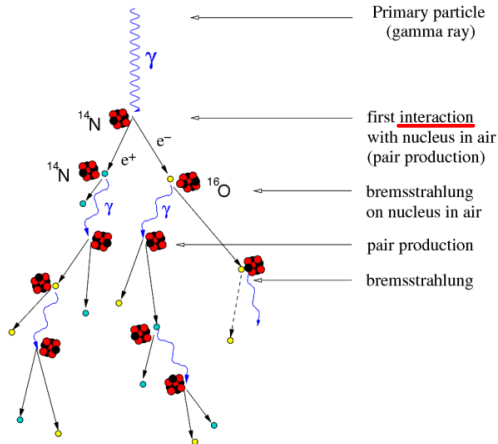
(particles, fluorescence- and Cherenkov-light, radio)

- **Key Observations (and their Interpretation)**

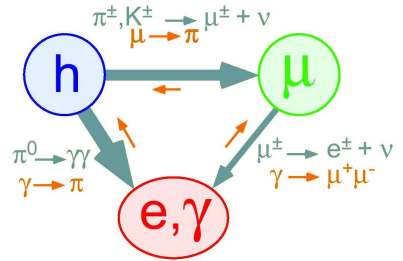
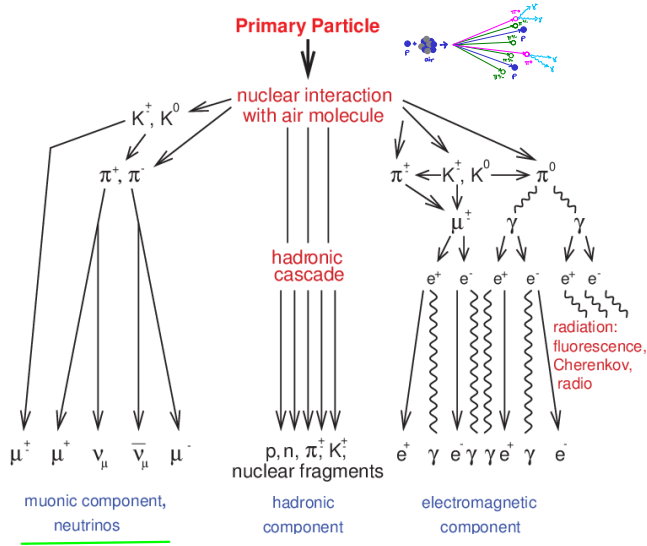
(anisotropies, mass, spectrum, Peters cycle, propagation, cosmic magnetic fields)



Particle Cascade in the Atmosphere / Air Shower



Particle Cascade in the Atmosphere / Air Shower



$\Rightarrow$ complicated coupled particle transport through atmosphere

$\Rightarrow$ numerical solutions or Monte Carlo

e.g. CORSIKA (dev. at IAP!)

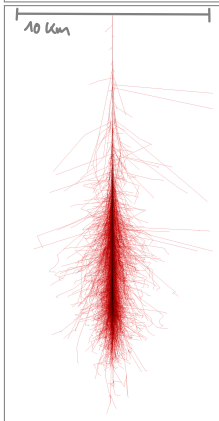
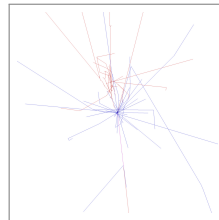
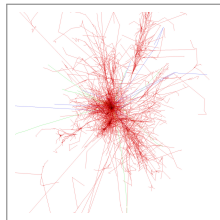
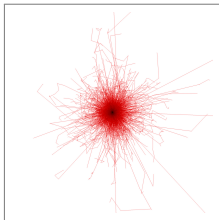
$$E = 10^{11} \text{ eV}$$

photon

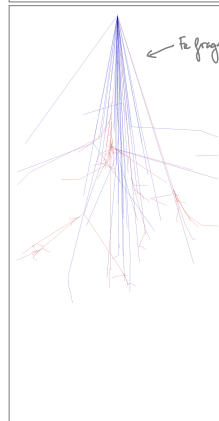
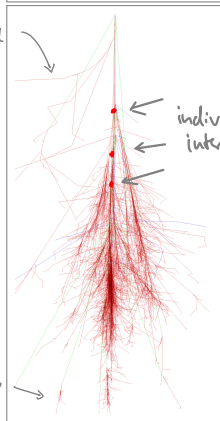
proton

iron

$\downarrow$
 $E_{\text{int}}!$
 $\Rightarrow E_{\text{kin}} = E_{\text{int}} - Mc^2$
 $\sim \begin{cases} 100 \text{ GeV} & \text{SiP} \\ 44 \text{ GeV} & \text{Fe} \end{cases}$
 $\Rightarrow E_{\text{kin}}/n < 1 \text{ GeV for Fe}$

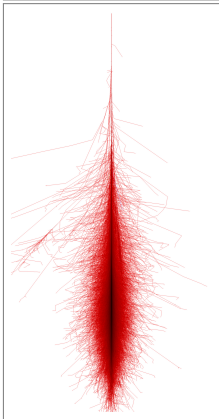
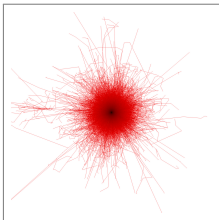


Earth magnetic field

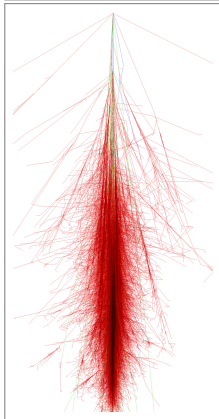
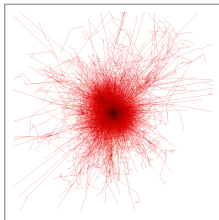


$$E = 10^{12} \text{ eV}$$

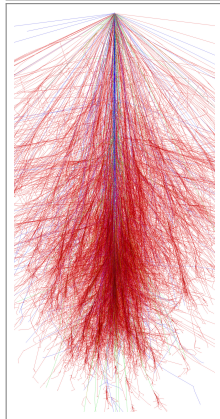
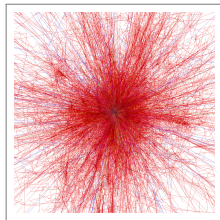
photon



proton

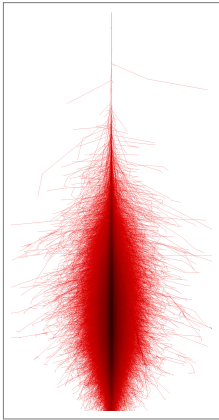
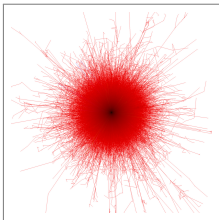


iron

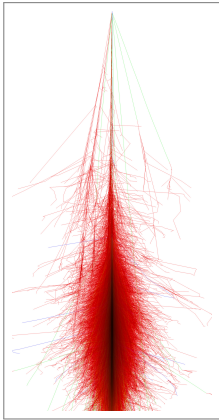
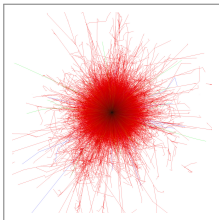


$$E = 10^{13} \text{ eV}$$

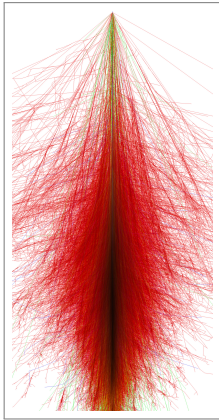
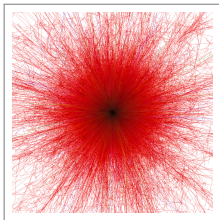
photon



proton

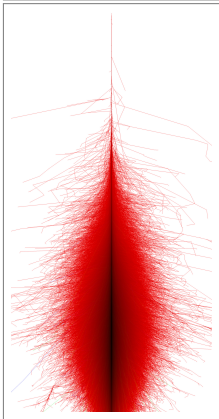
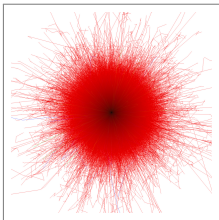


iron

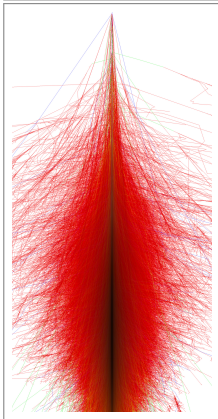
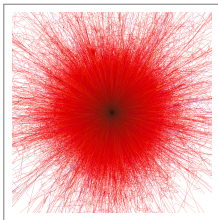


$$E = 10^{14} \text{ eV}$$

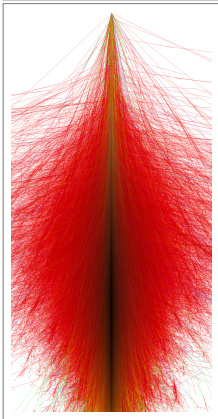
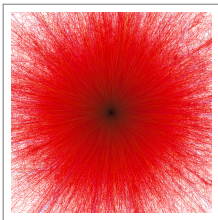
photon



proton

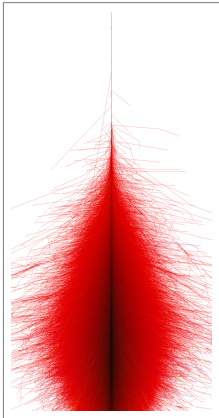
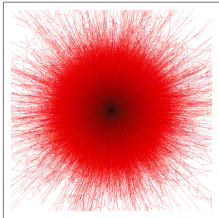


iron

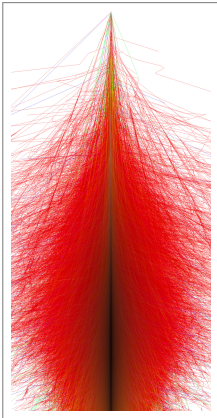
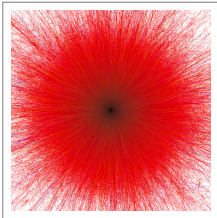


$$E = 10^{15} \text{ eV}$$

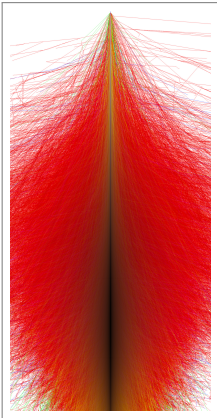
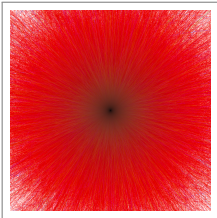
photon



proton



iron



Atmosphere

- height above sea level h
- air density $\rho(h)$
- vertical depth X_v

$$X_v = \int_h^{\infty} \rho(h') dh'$$

$$[X_v] = \text{g/cm}^2 \Rightarrow \text{"grammage"}$$

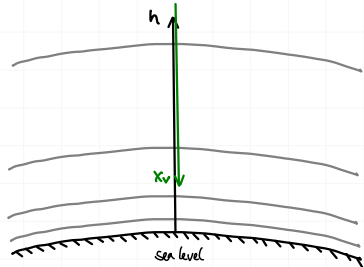
- isothermal atmosphere:

$$\rho(h) = \rho_0 e^{-h/h_0}$$

$$X_v = X_0 e^{-h/h_0}$$

- $X_0 \approx 1030 \text{ g/cm}^2$ at sea level

- scale height $h_0 \approx 8.4 \text{ km}$ at sea level, $\approx 6.4 \text{ km}$ high altitudes
above $h \approx 10 \text{ km}$



lateral spread
due to Coulomb
scattering

see lecture 2

h	X_v	$\rho(h)$			
altitude (km)	vertical depth (g/cm ²)	local density (10 ⁻³ g/cm ³)	Molière unit (m)	Cherenkov threshold (MeV)	Cherenkov angle (°)
40	3	3.8×10^{-3}	2.4×10^4	386	0.076
30	11.8	1.8×10^{-2}	5.1×10^3	176	0.17
20	55.8	8.8×10^{-2}	1.0×10^3	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1032	1.23	76	21	1.36

Atmosphere

- slant depth:

$$X = \int_e^{\infty} S(h(e')) de'$$

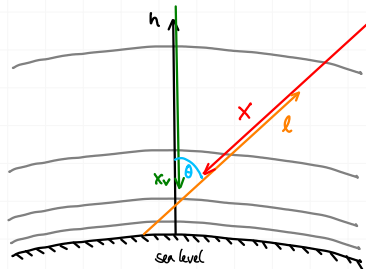
- zenith angle θ $h/e = \cos \theta$

- flat atmosphere approximation for $\theta \lesssim 65^\circ$

$$X = X_v / \cos \theta$$

- horizontal thickness of curved atmosphere:

$$X(\theta = 90^\circ) \approx 3.5 \cdot 10^4 \text{ g/cm}^2$$

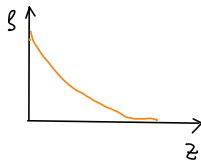
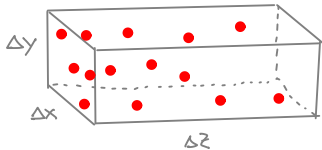


zenith angle degree	planar		spherical	
	distance km	slant depth g/cm ²	distance km	slant depth g/cm ²
0	112.8	1036.1	112.8	1036.1
30	130.3	1196.4	129.9	1196.0
45	159.6	1465.3	158.2	1463.7
60	225.7	2072.2	220.1	2065.3
70	329.9	3029.4	310.7	3003.9
80	649.8	5966.7	529.0	5765.9
85	1294.6	11887.9	770.9	10572.1
89	6465.0	59367.2	1098.3	25920.4
90	∞	∞	1204.4	36481.8

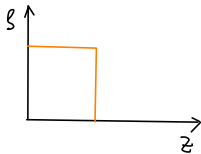
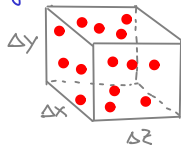
Table 1: Distances and slant depths in planar and spherical geometry, calculated with the Linsley parametrization of the U.S. standard atmosphere.

Column Depth X (cont.)

non-uniform density

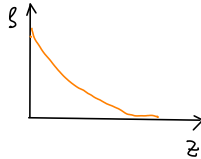
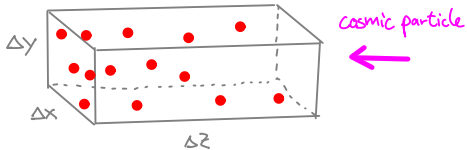


uniform density

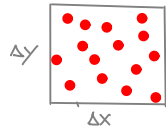


Column Depth X (cont.)

non-uniform density

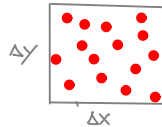
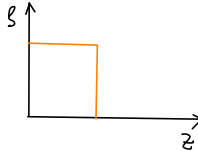
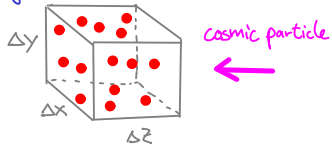


column number density



- N targets
- Size σ
- Mass m

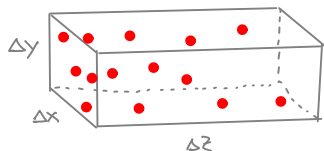
uniform density



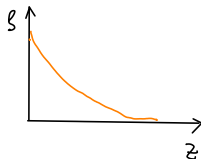
$$P(\text{interaction}) = \frac{S_{\text{target}}}{S_{\text{tot}}} = \frac{N \cdot \sigma}{\Delta x \Delta y}$$

Column Depth X (cont.)

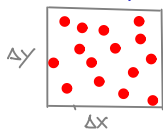
non-uniform density



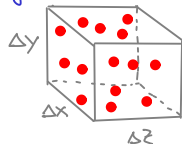
cosmic particle



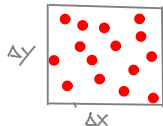
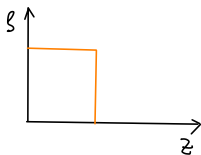
column number density



uniform density



cosmic particle



- N targets
- Size σ
- Mass m

$$P(\text{interaction}) = \frac{N_{\text{target}}}{N_{\text{tot}}} = \frac{N \cdot \sigma}{\Delta x \Delta y}$$

→ Column depth $X = \int s(z) dz$: mass per area $X = \frac{N \cdot m}{\Delta x \Delta y \Delta z} \Delta z$

→ interaction length $\lambda = \frac{m}{\sigma}$

$$\Rightarrow P(\text{int}) = 1 - e^{-X/\lambda} \approx \frac{X}{\lambda} = \frac{N \cdot \sigma}{\Delta x \Delta y}$$

Electromagnetic Interactions

energy loss

$$\left\langle -\frac{dE}{dx} \right\rangle_{\text{brms, pair}} = \frac{E}{X_0} \quad \Leftrightarrow E(x) = E_0 e^{-x/X_0}$$

radiation length:

$$X_0 \sim \left(\frac{Z^2}{A} S \right)^{-1}$$

critical energy:

$$\left\langle -\frac{dE}{dx} \right\rangle_{\text{brms}} \sim E$$

$$\left\langle \frac{dE}{dx} \right\rangle_{\text{ion}} \sim \text{const}$$

$$E_{\text{crit}} \text{ when } \left\langle -\frac{dE}{dx} \right\rangle_{\text{brms}} = \left\langle \frac{dE}{dx} \right\rangle_{\text{ion}}$$

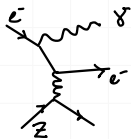
$\Rightarrow$ electron radiation length in air:

$$X_0^{\text{air}} = 36.6 \text{ g/cm}^2$$

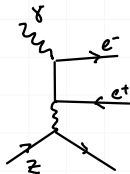
$\Rightarrow$ critical energy in air:

$$E_{\text{crit}}^{\text{air}} = 84 \text{ MeV}$$

interactions with nuclei of material (Z)



bremsstrahlung



pair production

Hadronic Interactions

- charge radius (e+p scattering):

$$r_p = 0.88 \cdot 10^{-15} \text{ m}$$

$$\rightarrow \sigma_{pp} \approx (2r_p)^2 \pi \approx 100 \text{ mb}$$

(b: "barn", $1b = 10^{-28} \text{ m}^2$)

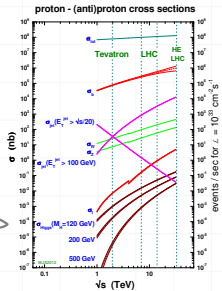
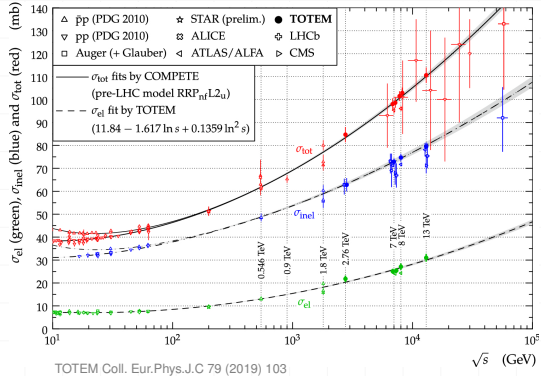
- inelastic cross section: $\sigma_{inel} = \sigma_{tot} - \sigma_{ela}$
 $\hat{=}$ particle production total elastic

$$\sigma_{inel} \approx 35 \text{ mb}$$

$$(10 \text{ GeV} < E_{\text{lab}} < 1 \text{ TeV})$$

- particle production: $p+p \rightarrow p+p + m \cdot \pi^{\pm} + n \cdot \pi^0$

- pion multiplicity: $m \approx 2 \cdot n$



(and $K^{\pm}, \Lambda, K^0, n$, Higgs ...)

but...

Hadronic Interactions

- interaction length: $j + \text{air} \rightarrow X$

$$\lambda_j = \ell_j S = \frac{S}{n_A \sigma_{j,\text{air}}} = \frac{\langle A \rangle m_p}{\sigma_{j,\text{air}}}$$

mass density $\downarrow$
 $\downarrow$ $\downarrow$ $\uparrow$ $\uparrow$
 $[\lambda] = \text{g/cm}^2$ $[e] = \text{cm}$ number density cross section

- typical values: @ 10 TeV

$$\lambda_N \approx 80 \text{ g/cm}^2 \quad p + \text{air} / n + \text{air}$$

$$\lambda_\pi \approx 100 \text{ g/cm}^2 \quad \pi + \text{air}$$

- average air mass: $\langle A \rangle = 14.6$ (78.09% N, 20.95% O, 0.93% Ar)

- nucleon + nucleus interactions:

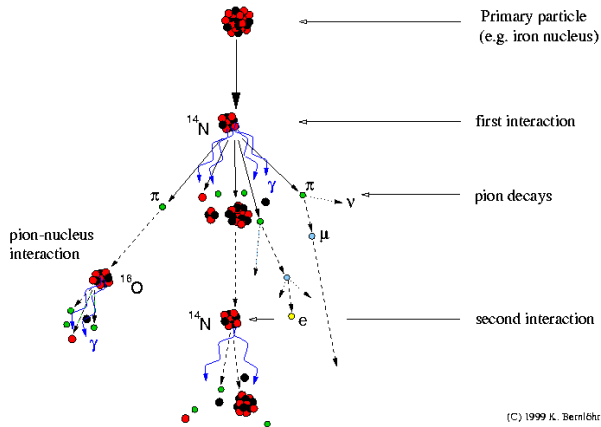
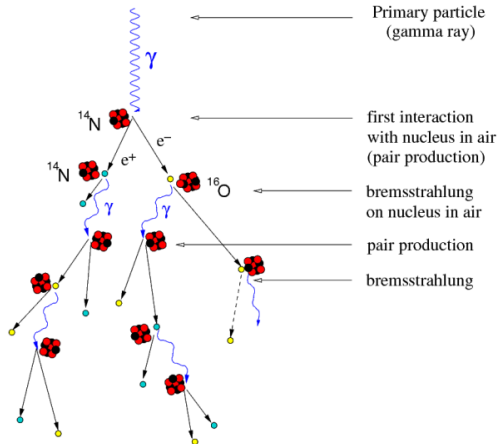
$$\sigma(p+A) \sim A^{2/3} \leftarrow \text{geometrical size of nucleus with } A \text{ spherically packed nucleons}$$

- nucleus + nucleus interactions:

$$\sigma(A_1 + A_2) \approx \pi R_0^2 (A_1^{1/3} + A_2^{1/3} - \delta)^2 \quad (\delta = 1.12, R_0 = 1.47 \text{ fm})$$

- Glauber model of $h+A$ scattering (see CRPP A6 and Glauber + Matthiae Nucl. Phys. B 21 (1970) 135)

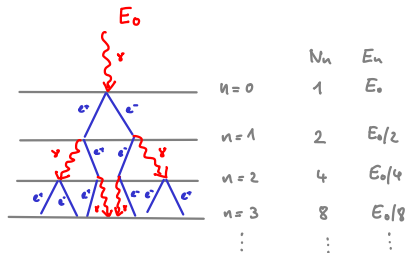
Particle Cascade in the Atmosphere / Air Shower



Photon-induced Shower

Heitler model

γ, e^-, e^+ shower



Carlson + Oppenheimer 1937, Heitler 1954

Photon-induced Shower

- radiation length X_0 in air: 37 g/cm^2

$$\left\langle -\frac{dE}{dx} \right\rangle_{\text{brems, pair}} = \frac{E}{X_0} \quad \Leftrightarrow \quad E(x) = E_0 \cdot e^{-x/X_0}$$

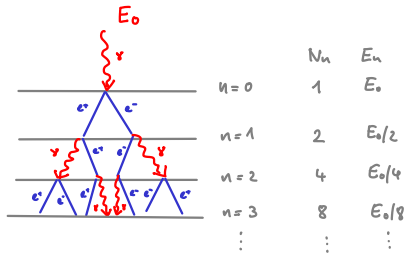
- splitting length $d = \ln 2 \cdot X_0$ $E(d) = E_0/2$

$$E_{i+1} \rightarrow E_i/2$$

$$N_{i+1} \rightarrow 2 \cdot N_i$$

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$$N_{i+1} \rightarrow 2 \cdot N_i$$

- after n splitting lengths:

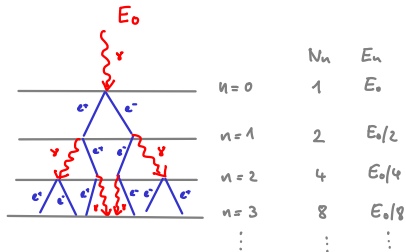
$$X_n = n \ln 2 X_0$$

$$N_n = 2^n = e^{x/X_0}$$

$$E_n = E_0/N_n$$

Heitler model

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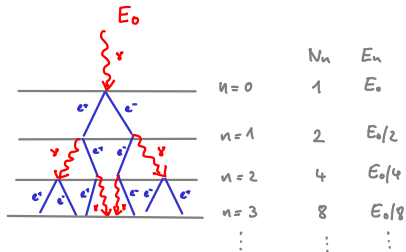
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Heitler model

γ, e^-, e^+ shower



Carlson + Oppenheimer 1937, Heitler 1954

Critical energy:

$$\frac{dE_{\text{rad}}}{dx}(E_{\text{crit}}) = \frac{dE_{\text{ion}}}{dx}(E_{\text{crit}})$$

in air: 84 MeV

- shower development stops when $E_n \leq E_{\text{crit}}$

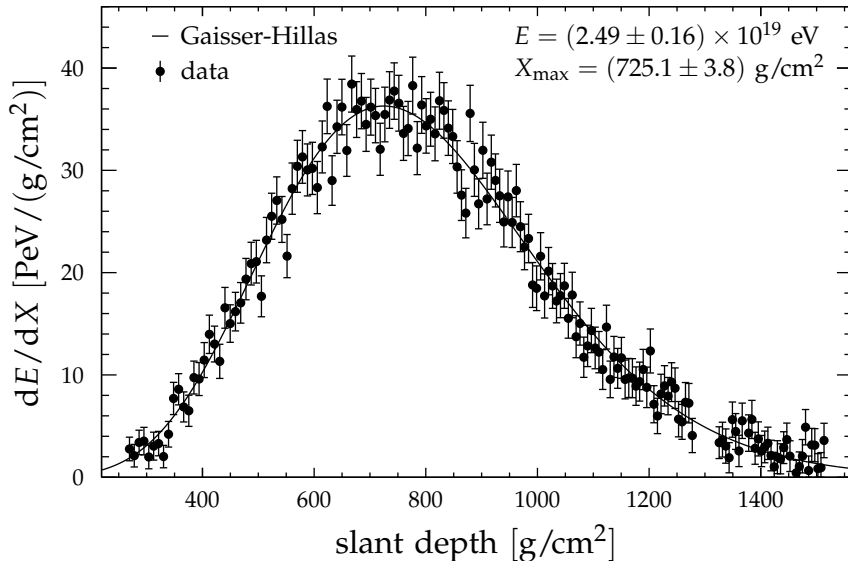
$$N_{\text{max}} = E_0/E_{\text{crit}} = 10^{11}$$

$$n_{\text{max}} = (\ln(E_0/E_{\text{crit}}))/\ln 2 = 37$$

$$X_{\text{max}} = X_0 (\ln(E_0/E_{\text{crit}})) = 340 \text{ g/cm}^2$$

$$\left. \begin{array}{l} N_{\text{max}} = 10^{11} \\ n_{\text{max}} = 37 \\ X_{\text{max}} = 340 \text{ g/cm}^2 \end{array} \right\} E_0 = 10^{19}$$

Example of a Measured Longitudinal Profile not a photon!



MIP: $dE/dX \sim 2 \text{ MeV/(g/cm}^2\text{)} \rightarrow N_{\text{max}} \sim 40 \times 10^{15} / 2 \times 10^6 = 2 \times 10^{10}$

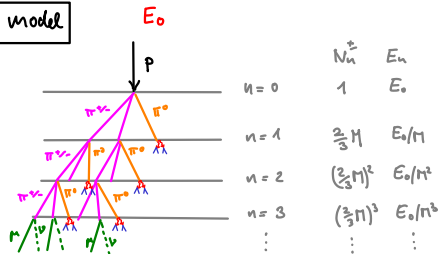
Proton-induced Shower

$$M = \pi_- + \pi_+ + \pi_0, \quad M_- \approx M_+$$

- production of M pions, $p + \text{air} \rightarrow \pi_- \cdot \pi^- + \pi_+ \cdot \pi^+ + \pi_0 \cdot \pi^0 + \dots$
(n : multiplicity)

Matthews-Heitler model

hadronic shower



J. Matthews APP 2005

Proton-induced Shower

$$M = \pi^- + \pi^+ + \pi^0, \quad M_- \approx M_+$$

• production of M pions, $p + \text{air} \rightarrow M_- \cdot \pi^- + M_+ \cdot \pi^+ + M_0 \cdot \pi^0 + \dots$

(n : multiplicity)

• interaction length λ_{int}

$$-E_{i+1} \rightarrow E_i/M, \quad E_n = E_0/M^n$$

• $\pi^\pm + \text{air} \rightarrow \frac{2}{3}M \cdot \pi^\pm + \frac{1}{3}M \pi^0$

re-interaction

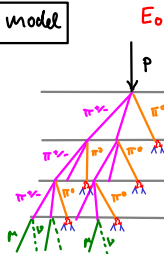
hadronic shower

decay
 $\pi^0 \rightarrow \gamma\gamma$

electromagnetic shower

Matthews-Heitler model

hadronic shower



n	$N_n^\pm$	E_n
$n=0$	1	E_0
$n=1$	$\frac{2}{3}M$	E_0/M
$n=2$	$(\frac{2}{3}M)^2$	E_0/M^2
$n=3$	$(\frac{2}{3}M)^3$	E_0/M^3
$\vdots$	$\vdots$	$\vdots$

J. Matthews APP 2005

Proton-induced Shower

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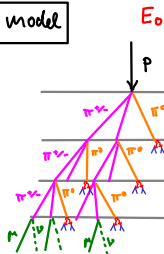
- hadronic cascade stops when $\lambda_{int} = \lambda_{dec}$ $\pi^+ \rightarrow \mu^+ \nu_\mu$
($\lambda_{dec} = 5.8 \text{ cm}$) $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$

$\rightarrow$ critical energy $E_\pi \approx 10 \text{ GeV} = E_n \rightarrow n_{crit} = (\ln(E_0/E_\pi)) / (\ln M)$

$$N_n = \left(\frac{2}{3}M\right)^n$$

Matthews-Heitler model

hadronic shower



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$\vdots$	$\vdots$	$\vdots$

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Proton-induced Shower

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- production of M pions, $p + \text{air} \rightarrow M_- \cdot \pi^- + M_+ \cdot \pi^+ + M_0 \cdot \pi^0 + \dots$

(M : multiplicity)

- interaction length λ_{int}

$$-E_{i+1} \rightarrow E_i/M, \quad E_n = E_0/M^n$$

$$\pi^\pm + \text{air} \rightarrow \frac{2}{3}M \cdot \pi^\pm + \frac{1}{3}M \pi^0$$

re-interaction

hadronic shower

decay
 $\pi^0 \rightarrow \gamma\gamma$

electromagnetic shower

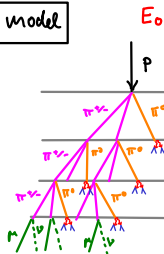
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($\lambda_{\text{dec}} = 5.8 \text{ cm}$) $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$

$\rightarrow$ critical energy $\epsilon_\pi \approx 10 \text{ GeV} = E_n \rightarrow n_{\text{crit}} = (\ln(E_0/\epsilon_\pi))/(\ln M)$

$$N_n = \left(\frac{2}{3}M\right)^n$$

Matthews-Heitler model

hadronic shower



	$N_n^\pm$	E_n
$n=0$	1	E_0
$n=1$	$\frac{2}{3}M$	E_0/M
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$n=3$	$(\frac{2}{3}M)^3$	E_0/M^3
$\vdots$	$\vdots$	$\vdots$

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$$N_M = N_{\text{crit}} = \left(\frac{E_0}{\epsilon_\pi}\right)^{\frac{1}{\ln M}}$$

With $\beta = \frac{\ln \frac{2}{3}M}{\ln M} = 1 - \frac{\ln \frac{3}{2}}{\ln M} = 1 - \frac{\ln \frac{3}{2}}{\ln 50} \approx 1 - \frac{0.18}{\ln 50}$
 ≈ 0.9 for $M=50$

e.g. $N_M \approx 10^8$ for $E_0 = 10^{15} \text{ eV}$ and $M=50$

Proton-induced Shower

estimate of shower maximum:

- photons produced in π^0 decays after first interaction:

$$\frac{1}{3}M \pi^0 \text{ with } E = E_0/M \Rightarrow 2 \cdot \frac{1}{3}M \text{ photons with } E_\gamma = E_0/M/2$$

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- $2 \cdot \frac{1}{3} M$ electromagnetic showers starting at $\langle x_1 \rangle = \lambda_p$

$$\langle X_{\max}^p \rangle = \lambda_p + X_{\max}^{\gamma} \left(E = \frac{E_0}{2M} \right)$$

$$\Rightarrow \boxed{\langle X_{\max}^p \rangle = \lambda_p + X_0 \ln \left(\frac{E_0}{2 \cdot M \cdot \epsilon_c} \right)}$$

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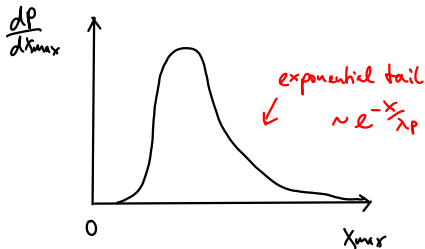
- $2 \cdot \frac{1}{3} M$ electromagnetic showers starting at $\langle x_s \rangle = \lambda_p$

$$\langle X_{\max}^p \rangle = \lambda_p + X_{\max}^e (E = \frac{E_0}{2M})$$

$$\Rightarrow \boxed{\langle X_{\max}^p \rangle = \lambda_p + X_0 \ln \left(\frac{E_0}{2 \cdot M \cdot E_c} \right)}$$

$\Rightarrow X_{\max}$ distribution

$$\frac{dP}{dX_{\max}} = \underbrace{\frac{dP}{dX_1}}_{\text{1st interaction}} \otimes \underbrace{\frac{dP}{d\Delta X}}_{\text{shower development}} \quad \begin{matrix} \text{exp} & \approx \text{gauss} \end{matrix} \quad \Delta X = X_{\max} - X_1$$

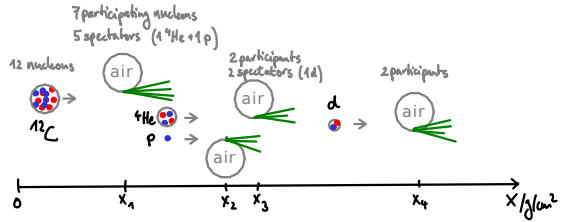


$\Rightarrow$ measurement of σ_{p+air} @ $\sqrt{s} \approx 100 \text{ TeV} \gg \sqrt{s}_{LHC}$

Nucleus-induced Shower

Superposition model $(E, A) + \text{air} \rightarrow X \cong A \cdot (E/A, 1) + \text{air} \rightarrow X$

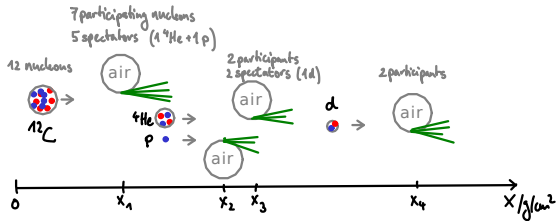
but e.g.:



Nucleus-induced Shower

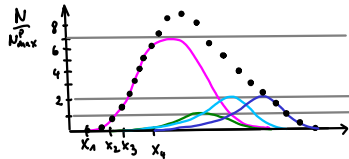
Superposition model $(E, A) + \text{air} \rightarrow X \hat{=} A \cdot (E/A, 1) + \text{air} \rightarrow X$

but e.g.:



$$\rightarrow P(X_{i+1} - X_i) \sim e^{-\frac{X_{i+1} - X_i}{\lambda_{A\text{air}}}}$$

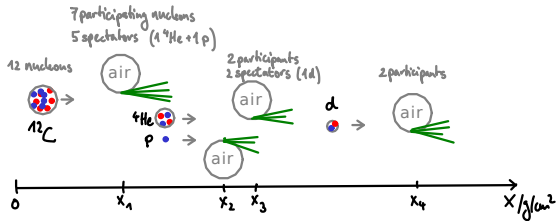
$\rightarrow$ superposition of 12 N+air showers from 4 A_{air} +air interactions: 7N@ x_1 , 1N@ x_2 , 2N@ x_3 , 2N@ x_4



Nucleus-induced Shower

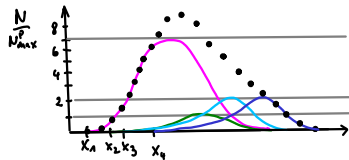
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$\rightarrow$ superposition of 12 N+air showers from 4 A_n+air interactions: 7N@x₁, 1N@x₂, 2N@x₃, 2N@x₄



Superposition theorem J. Engel et al. PRD 1992

if average number of participating nucleons in projectile A+air interactions

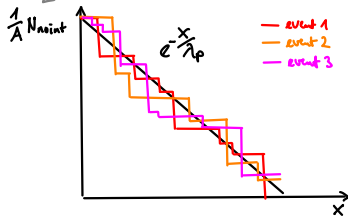
$$\langle N_A \rangle = A \frac{\lambda_A}{\lambda_p} \quad \langle N_A \rangle = \sum n P(n)$$

then probability of depth of interaction of nucleons

$$\frac{dP_A}{dX} = \frac{1}{\lambda_p} e^{-X/\lambda_p}$$

irrespective of fragmentation of spectator nucleus

number of nucleons that have not yet interacted $\frac{1}{A} N_{\text{point}} = 1 - \int_0^X \frac{dP_A}{dX'} dX' = \exp(-X/\lambda_p)$



Nucleus-induced Shower

• number of muons: $N_\mu = A \cdot \left(\frac{E_0/A}{\varepsilon_\pi} \right)^\beta = \left(\frac{E_0}{\varepsilon_\pi} \right)^\beta A^{1-\beta}$

e.g. $M=50, \beta=0.9$

$\rightarrow N_\mu(56)/N_\mu(1) = 56^{0.1} = 1.5$

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 $\rightarrow \underline{N_\mu(56)/N_\mu(1) = 56^{0.1} = 1.5}$
- energy in γ, e^+, e^- : $E_{em} = E_0 - N_\mu \cdot \varepsilon_\pi$ e.g. $E_0 = 10^{20} \text{ eV}, M=50, A=1 \Rightarrow \underline{E_{em}/E_0 \approx 91\%}$
 $A=56 \Rightarrow \underline{E_{em}/E_0 \approx 86\%}$

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- number of γ, e^+, e^- : $N_{em} = E_{em}/\varepsilon_{crit} = \frac{E_{em}}{E_0} \cdot N_{em}^g$
- shower maximum from 1st interaction:

$$\underline{X_{max}^A = \lambda_p + X_0 \ln\left(\frac{E_0}{2 \cdot A \cdot M \cdot \varepsilon_c}\right)}$$

$$A \cdot \frac{1}{3} M \pi^0 \text{ with } E = E_0/A/M \Rightarrow 2 \cdot A \cdot \frac{1}{3} M \gamma\text{'s with } E_\gamma = E_0/A/M/2$$

$$\underline{X_{max}^A(E) = X_{max}^P(E/A)}$$

Nucleus-induced Shower

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$A \cdot \frac{1}{3} M \pi^\circ$ with $E = E_0/A/M \Rightarrow 2 \cdot A \cdot \frac{1}{3} M \gamma's$ with $E_\gamma = E_0/A/M/2$

$$\underline{X_{max}^A(E) = X_{max}^P(E/A)}$$

how to measure the CR mass + energy with air showers

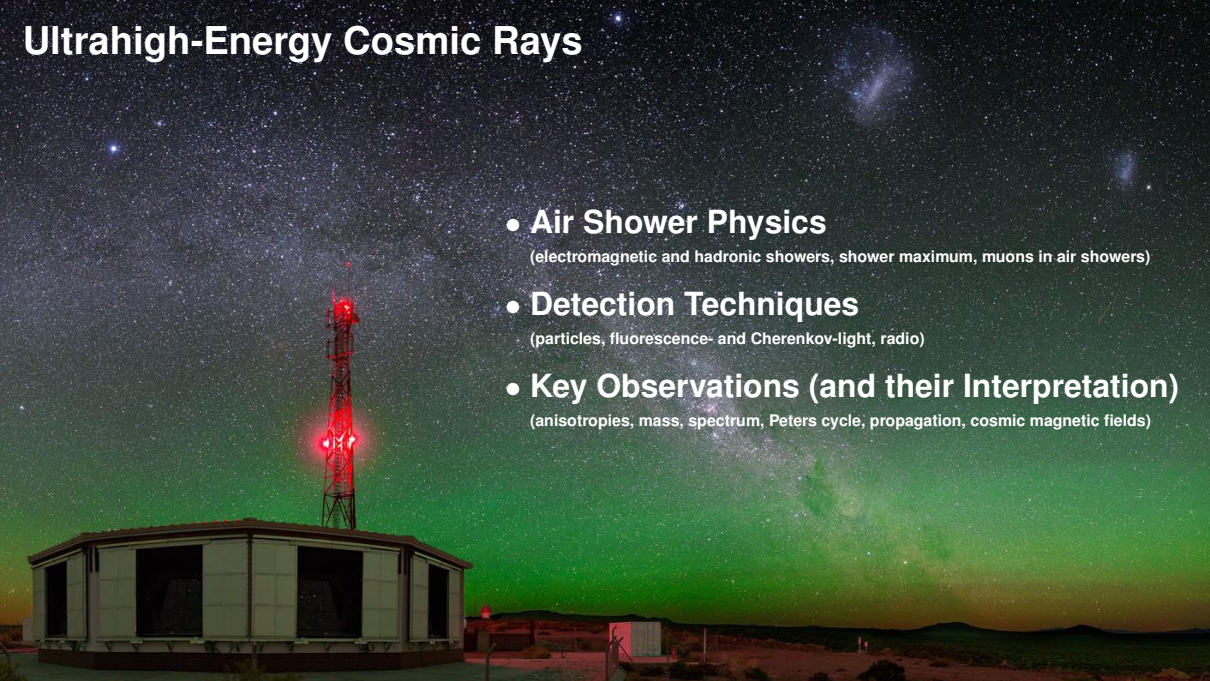
measurement	primary CR	detector
N_e, N_μ	$\leftrightarrow E_0, A$	SD
X_{max}, E_{em}	$\leftrightarrow E_0, A$	FD/CD
E_{em}, N_μ	$\leftrightarrow E_0, A$	RD+SD
$N_e, N_\mu, X_{max}, E_{em}$	$\leftrightarrow E_0, A$	SD+FD

} "hybrid"

overconstrained $\Rightarrow$ check hadronic interaction models

(SD: surface detector (particles)
 FD: fluorescence detector
 CD: Cherenkov detector
 RD: radio detector)

Ultra-high-Energy Cosmic Rays

A night sky with a green aurora borealis and a radio telescope structure. The sky is dark with many stars and a bright green aurora borealis. A radio telescope structure is visible on the left, with a red light at the top. The ground is dark and flat.

- **Air Shower Physics**

(electromagnetic and hadronic showers, shower maximum, muons in air showers)

- **Detection Techniques**

(particles, fluorescence- and Cherenkov-light, radio)

- **Key Observations (and their Interpretation)**

(anisotropies, mass, spectrum, Peters cycle, propagation, cosmic magnetic fields)